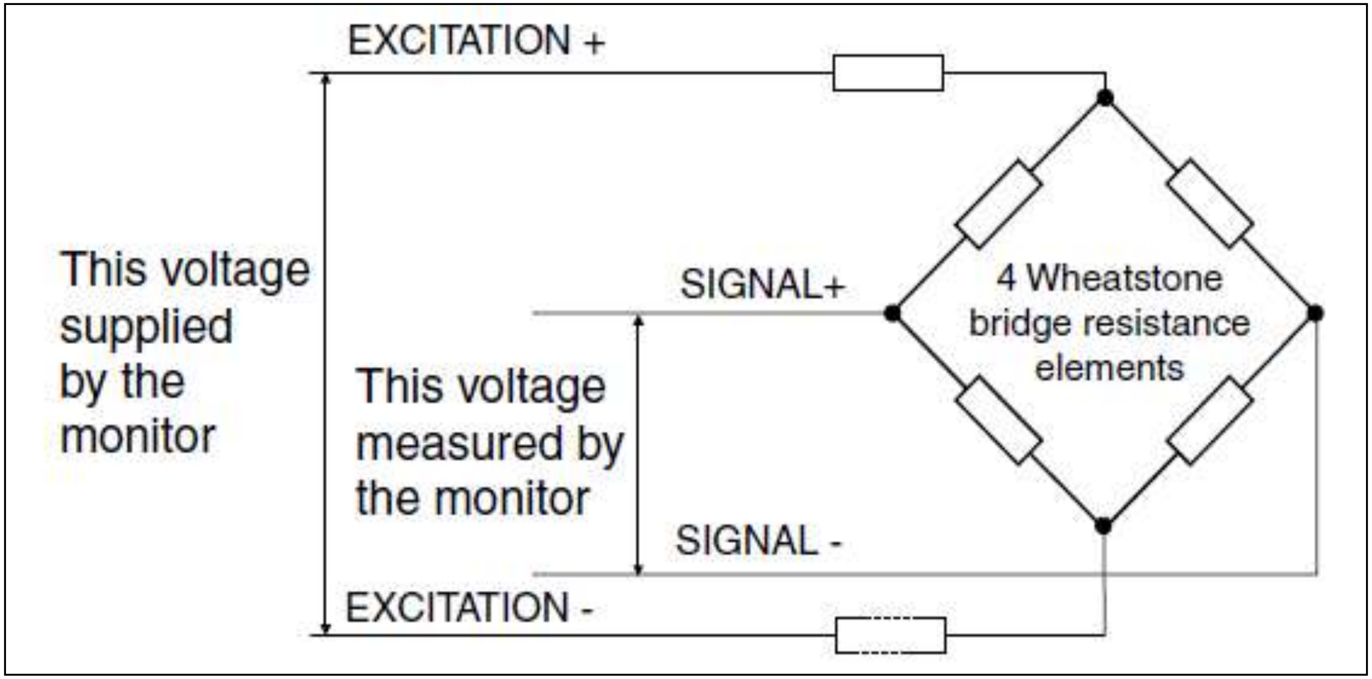
LOAD CELL

- Load cells, transducers that convert a force into an electric signal, are used in a wide variety of force and weight monitoring applications in industries ranging from industrial automation, oil and gas, transportation, and aerospace etc.

- Honeywell offers load cells in many form factors, such as low profile/pancake, miniature and subminiature, canister, donut, in-line/rod end, beam style, and S and Z beam style.
- Packaging options include stainless steel, carbon steel, and aluminum.

- **Basic load cell system**
- **It consists of a 4 wire load cell connected to a load cell monitor.**
- **The monitor supplies a voltage called the excitation voltage to the load cell and the load cell returns a millivolt (mV) signal back to the monitor.**
- **The mV signal changes with the load experienced by the load cell and this mV signal is used to determine the display value on the monitor.**


EXCITATION +
This voltage supplied by the monitor
SIGNAL+
4 Wheatstone bridge resistance elements
This voltage measured by the monitor
SIGNAL -
EXCITATION -

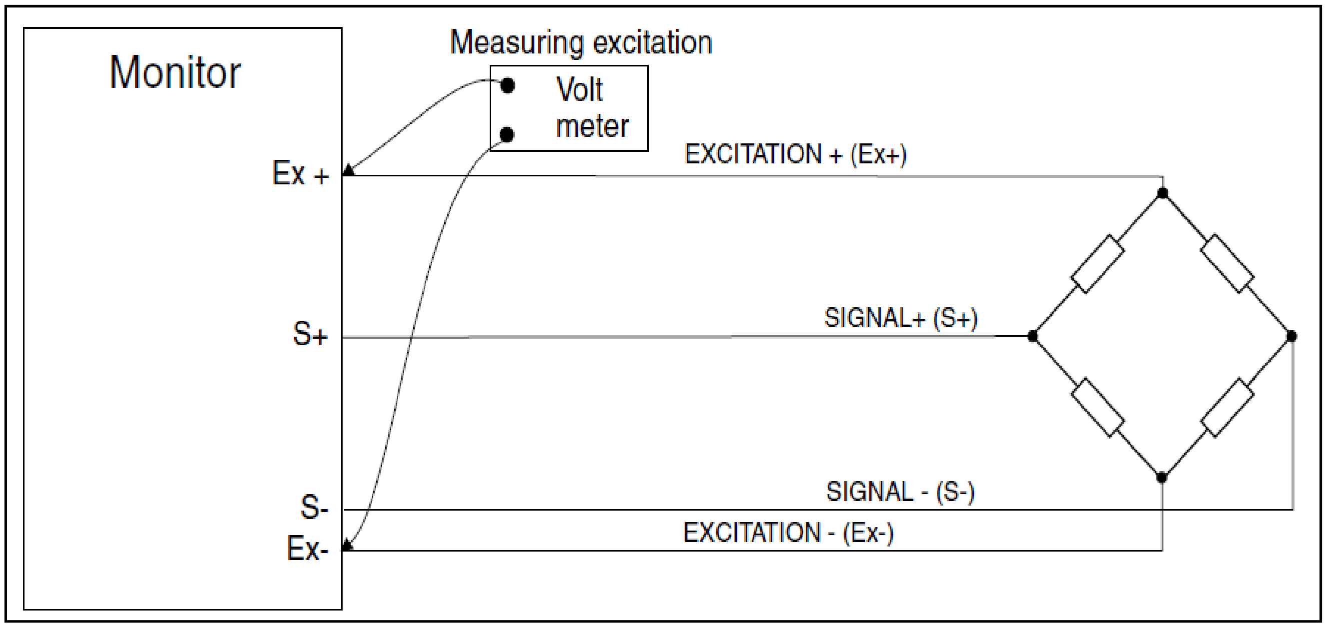
- **Basic load cell resistance checks**
- **There is no standard wiring colour code for load cells, so need to know from data supplied with the cell which wires are the excitation wires and which wires are the signal wires.**
- **If you do not have this information, then the 4 resistance elements which make up the Wheatstone bridge there are commonly one or two resistors in the excitation lines.**

- This means that the resistance across the excitation wires is usually the highest resistance measured across any two wires.
- The resistances also vary between manufacturers and load cell types, input and output resistance values are often provided in the data supplied with the load cell.

- To test the cell obtain a meter capable of measuring Ohms, measure across each pair of wires in turn and record the results.
- The load cell must be completely disconnected from the monitor and at no load when these tests are made.

- When installing a load cell system the usual installation procedure would be:
- 1. Install load cell in position
- 2. Connect excitation and signal wires to monitor
- 3. Calibrate monitor to read the load in the units required e.g. kg or tones (consult monitor instruction manual for calibration procedures)
- 4. Check monitor is reading correctly over a range of values

- **Giving unstable readings make the following measurements:**
- **1. Using a voltmeter measure and record the excitation voltage**
- **2. Using a mV meter measure and record the signal voltage at the lowest load**
- **3. Using a mV meter measure and record the signal voltage**
- **4. Obtain the mV/V output figure from the load cell data supplied**



Monitor
Measuring excitation
Volt
meter
Ex +
EXCITATION + (Ex+)
S+
SIGNAL+ (S+)
S-
SIGNAL - (S-)
Ex-
EXCITATION - (Ex-)

- **Testing the monitor**
- **If the resistance checks and mV output checks from the load cell appear to be correct but the system is not responding correctly to a change in load the load cell monitor needs to be checked using a load cell simulator.**

- **Some common problems which may cause difficulty when installing a system:**
- **Incorrect wiring of load cell to monitor**
- **Friction or restriction of load cell movement**
- **Over stressed load cell**
- **Moisture in the load cell or wiring**

- High resistance cable joins
- Incorrect mounting of the load cell
- Electrical damage to load
- Non linearity of display reading

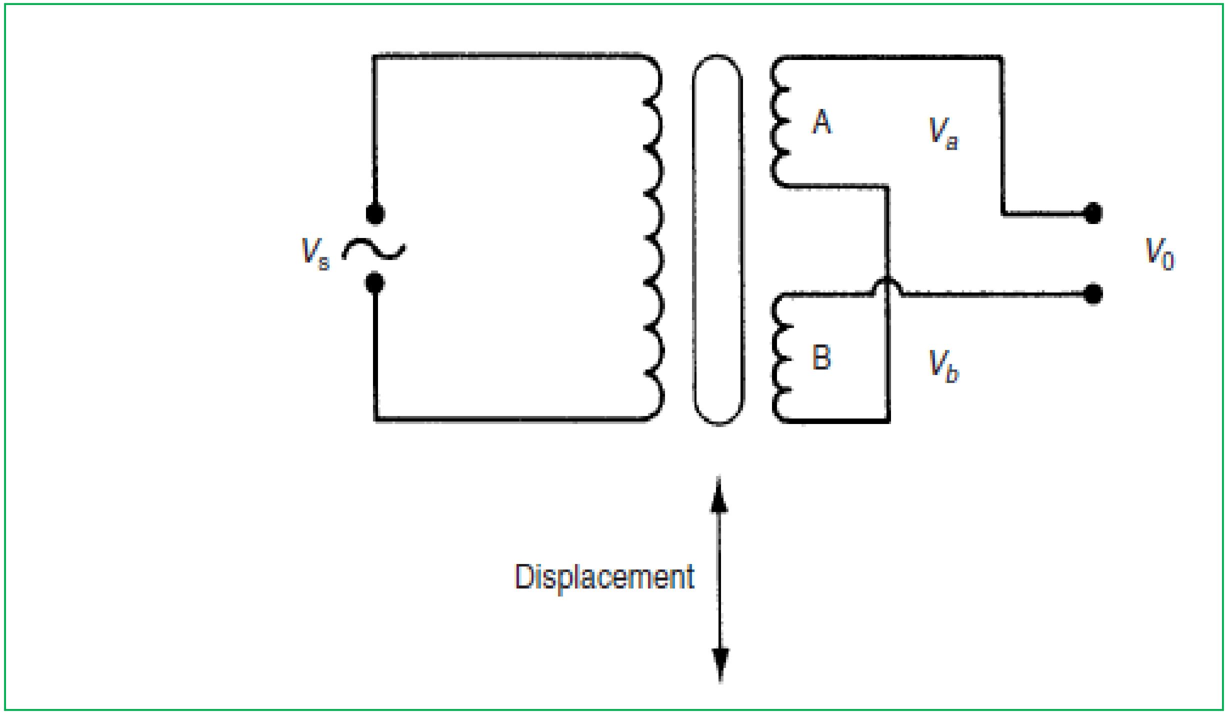
Load cell specification terms

- Accuracy
- Rated Capacity
- Compensated temperature range
- Environmental protection
- Excitation
- Hysteresis
- Input resistance
- Insulation resistance

- **Mechanical failure**
- **Rated output**
- **Output resistance**
- **Repeatability**
- **Safe load**
- **Temperature effect on span**
- **Temperature effect on zero**
- **Zero balance /Zero return (creep)**

LINEAR VARIABLE DIFFERENTIAL TRANSFORMER

- LVDT, consists of a transformer with a single primary winding and two secondary windings connected in the series opposing manner shown in Figure.
- The object whose translational displacement is to be measured is physically attached to the central iron core of the transformer, so that all motions of the body are transferred to the core.



The Linear variable differential transformer(LVDT)

- The only moving part in an LVDT is the central iron core.
- As the core is only moving in the air gap between the windings, there is no friction or wear during operation.
- For this reason, the instrument is a very popular one for measuring linear displacements and has a quoted life expectancy of 200 years.

MEASUREMENT OF PRESSURE

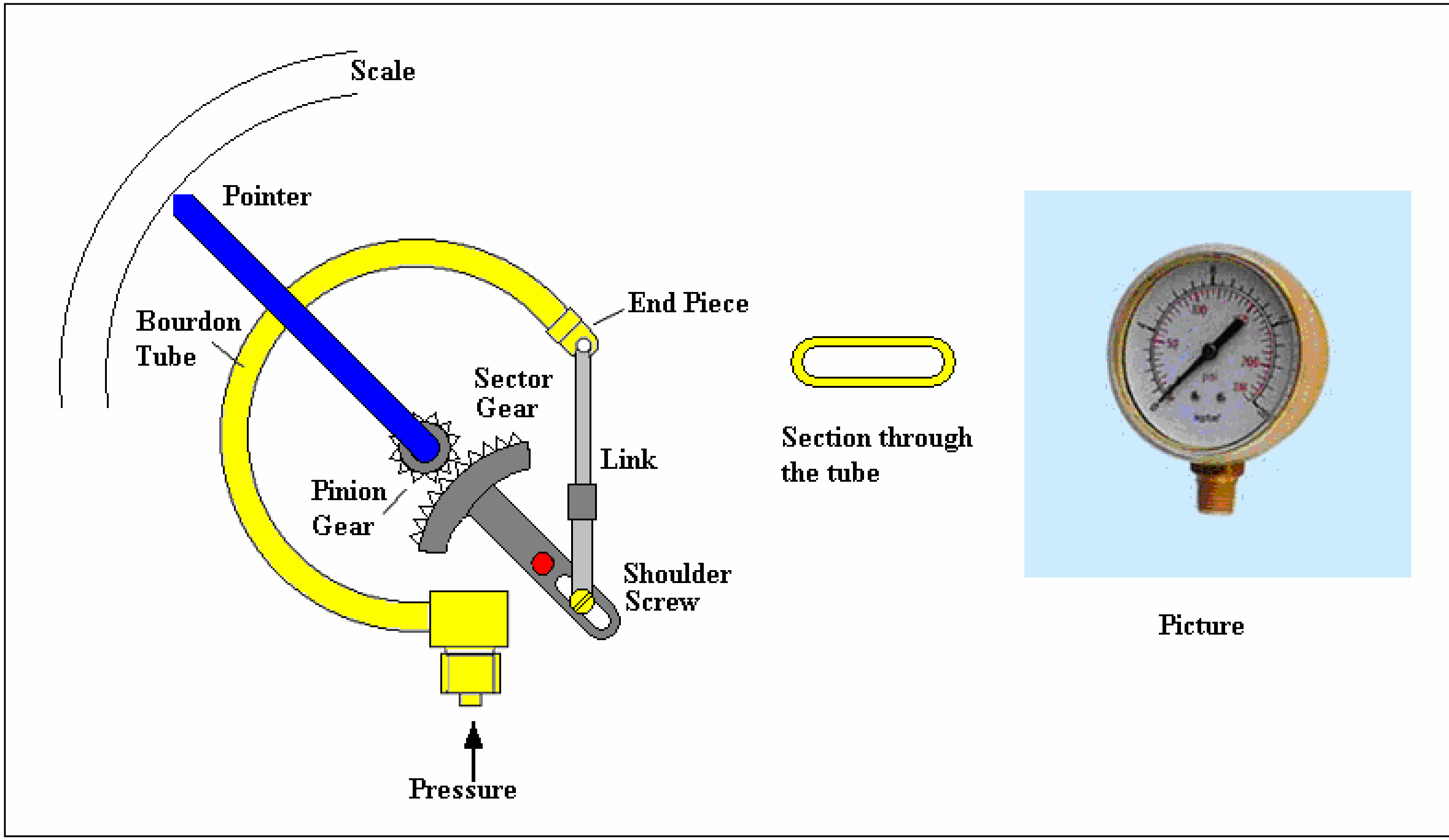
- Pressure is probably one of the most commonly measured variables in the power plant.
- It includes the measurement of steam pressure; feed water pressure, condenser pressure, lubricating oil pressure and many more.
- Pressure is actually the measurement of force acting on area of surface. Pressure = Force/Area.

- The units of measurement are either in pounds per square inch (psi) in British units or Pascals (Pa) in metric.
- A gauge pressure device will indicate zero pressure when bled down to atmospheric pressure (i.e., gauge pressure is referenced to atmospheric pressure).
- Gauge pressure is denoted by a (g) at the end of the pressure unit [e.g., kPa (g)].

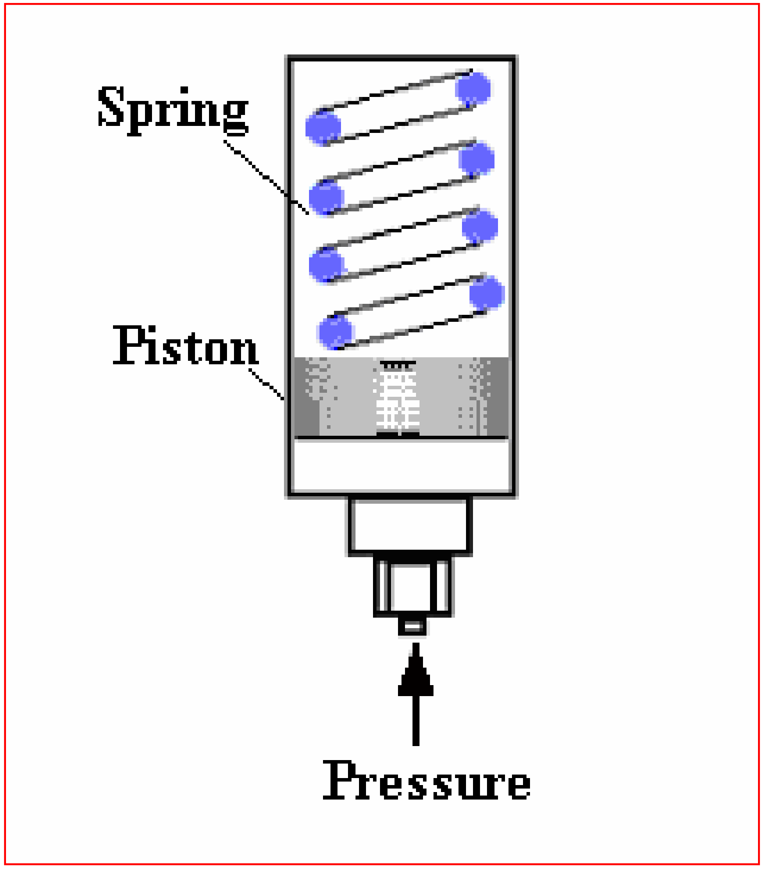
- Absolute pressure includes the effect of atmospheric pressure with the gauge pressure.
- Absolute Pressure = Gauge Pressure + Atmospheric Pressure
- Absolute measurements tend to be used where pressures are below atmosphere.
- Typically this is around the condenser and vacuum building.

- To accomplish this, most pressure sensors translate pressure into physical motion that is in proportion to the applied pressure.
- The most common pressure sensors or primary pressure elements are included:
- Diaphragms, pressure bellows, bourdon tubes and pressure capsules.

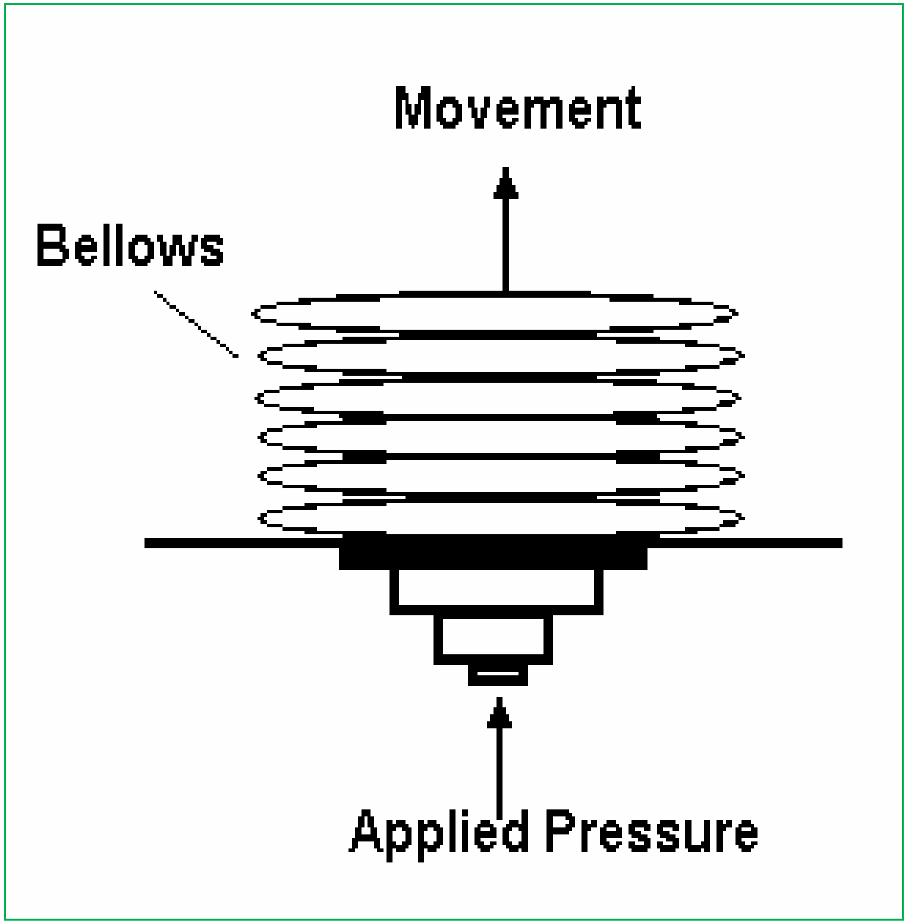
- With these pressure sensors, physical motion is proportional to the applied pressure within the operating range.
- The term differential pressure is often used.
- This term refers to the difference in pressure between two quantities, systems or devices.



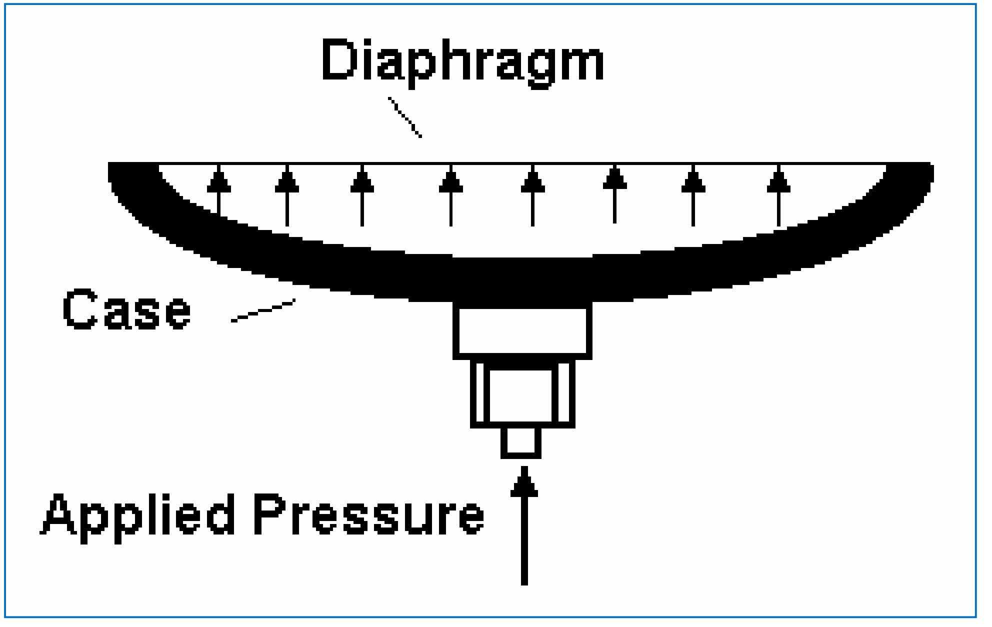
Bourdon Tube



Spring and Piston



Bellows and capsules



Diaphragm

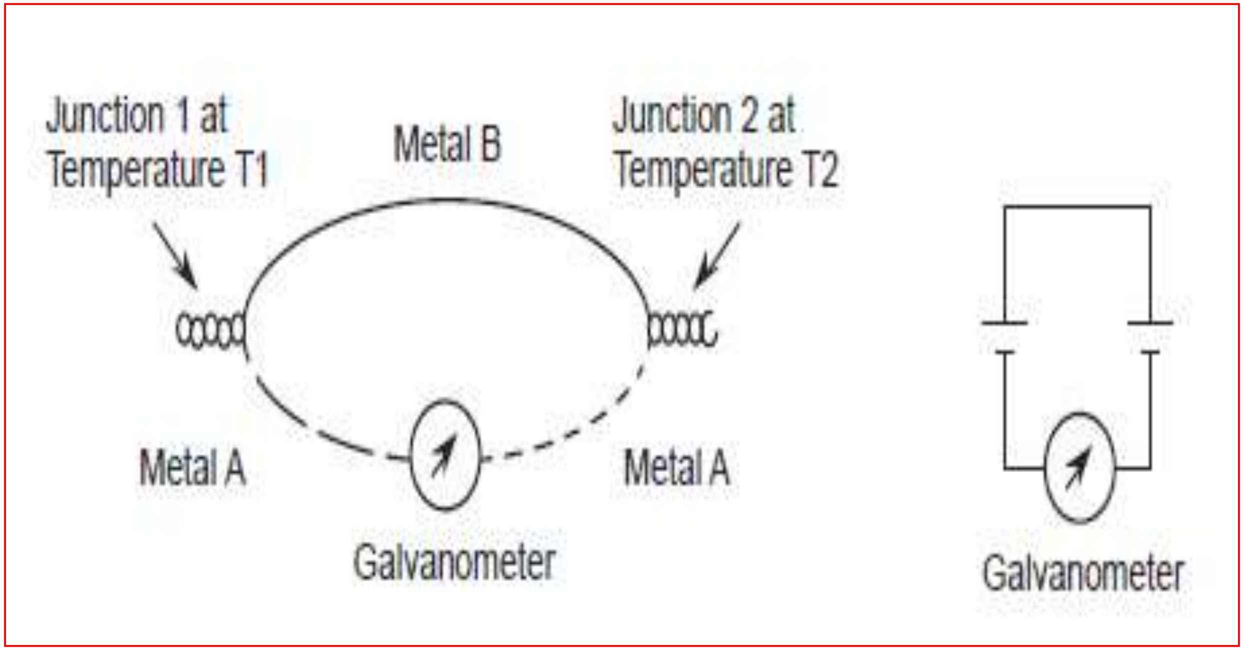
TEMPERATURE MEASUREMENT

- **Every aspect of our lives, both at home and at work, is influenced by temperature.**
- **Temperature measuring devices have been in existence for centuries.**
- **The age-old mercury in glass thermometer is still used today and why not?**

- Its operation was based on the temperature expansion of fluids (mercury or alcohol).
- As the temperature increased the fluid in a small reservoir or bulb expanded and a small column of the fluid was forced up a tube.
- You will find the same theory is used in many modern thermostats today.
- In this module some temperature measuring devices commonly found in a generating station.

- These include thermocouples, thermostats and resistive temperature devices.
- Thermocouples (TC) and resistive temperature devices (RTD) are generally connected to control logic or instrumentation for continuous monitoring of temperature.
- Thermostats are used for direct positive control of the temperature of a system within preset limits.

- **Thermocouple (TC)**
- **A thermocouple consists of two pieces of dissimilar metals with their ends joined together (by twisting, soldering or welding).**
- **When heat is applied to the junction, a voltage, in the range of milli-volts (mV), is generated.**
- **A thermocouple is therefore said to be self-powered.**



A Thermocouple Circuit

- **In order to use a thermocouple to measure process temperature, one end of the thermocouple has to be kept in contact with the process while the other end has to be kept at a constant temperature.**
- **The end that is in contact with the process is called the hot or measurement junction.**
- **The one that is kept at constant temperature is called cold or reference junction.**

Advantages:

- Thermocouples are used on most transformers. The hot junction is inside the transformer oil and the cold junction at the meter mounted.
- In general, thermocouples are used exclusively around the turbine hall because of their rugged construction and low cost.
- A thermocouple is capable of measuring a wider temperature range than an RTD.

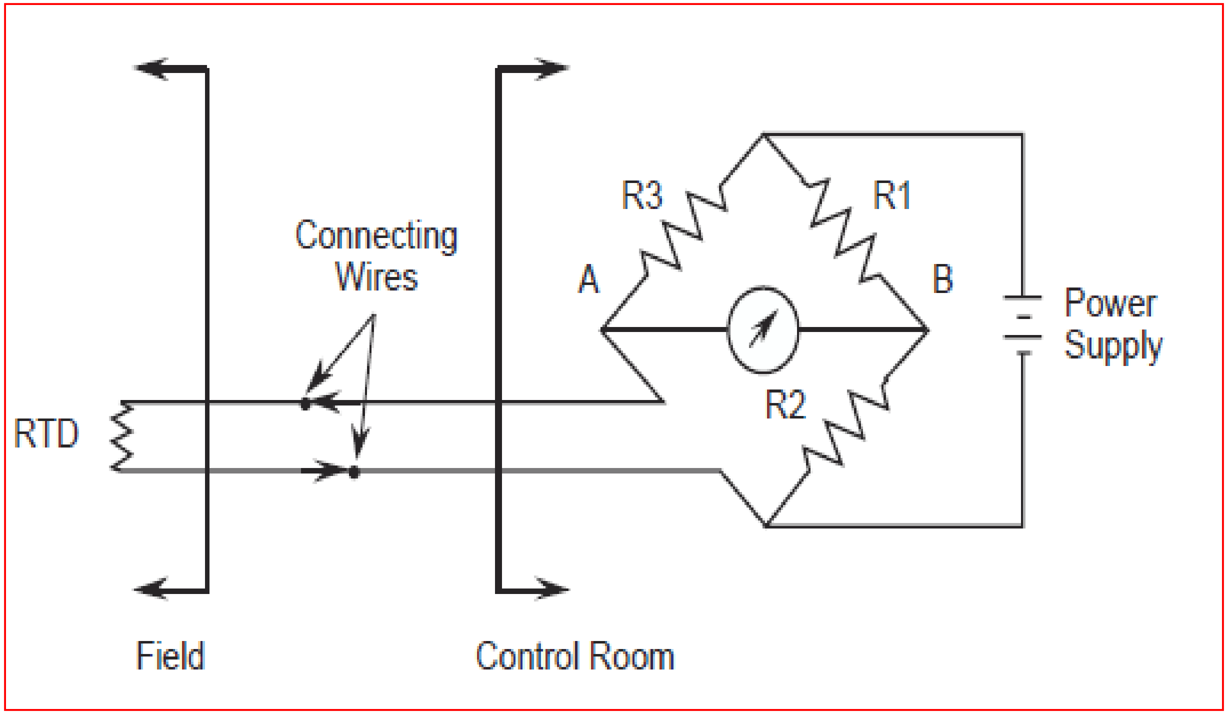
Disadvantages:

- Thermocouples are not used in areas where high radiation fields are present.
- Thermocouples are slower in response than RTDs
- If the control logic is remotely located and temperature transmitters are used, a power supply failure will of course cause faulty readings.

- **Resistance Temperature Detector (RTD)**
- **For most metals the change in electrical resistance is directly proportional to its change in temperature and is linear over a range of temperatures.**
- **This constant factor called the temperature coefficient of electrical resistance (TCR)on the basis of RTD.**
- **The RTD can actually be regarded as a high precision wire wound resistor whose resistance varies with temperature.**

- **Several different pure metals (such as platinum, nickel and copper) can be used in the manufacture of an RTD.**
- **A typical RTD probe contains a coil of very fine metal wire, allowing for a large resistance change without a great space requirement.**
- **Usually, platinum RTDs are used as process temperature monitors because of their accuracy and linearity.**

- To detect the small variations of resistance of the RTD, a temperature transmitter in the form of a Wheatstone bridge is generally used.
- The circuit compares the RTD value with three known and highly accurate resistors.
- A Wheatstone bridge consisting of an RTD, three resistors, a voltmeter and a voltage source is illustrated in Figure.



RTD using a Wheatstone Bridge

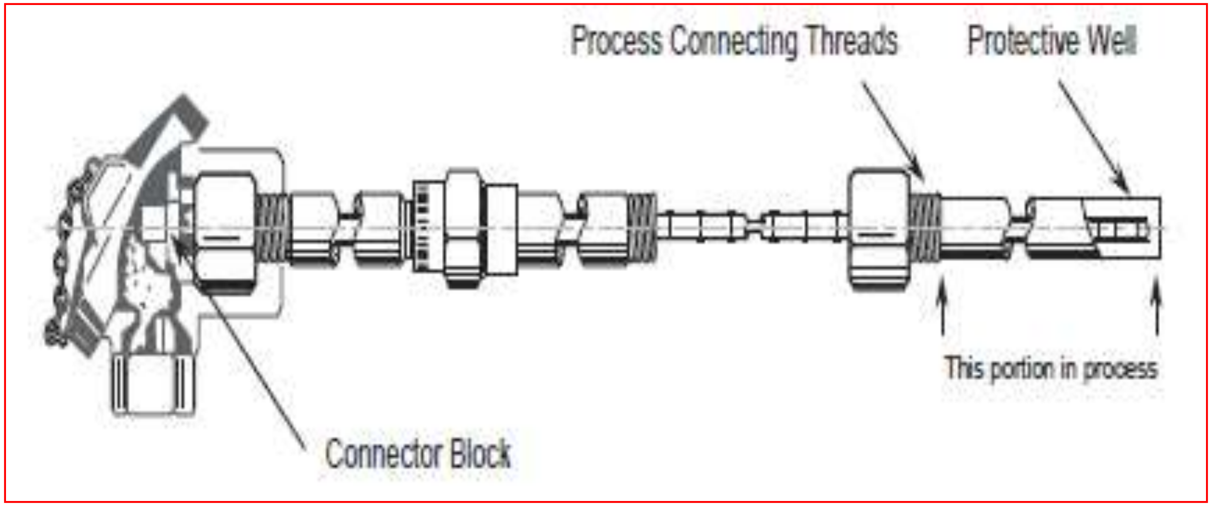
Advantages:

- The response time compared to thermocouples is very fast.
- An RTD will not experience drift problems because it is not self powered.
- Within its range it is more accurate and has higher sensitivity than a thermocouple.
- The RTD does not require special extension cable.
- Radioactive radiation has minimal effect on RTDs.

Disadvantages:

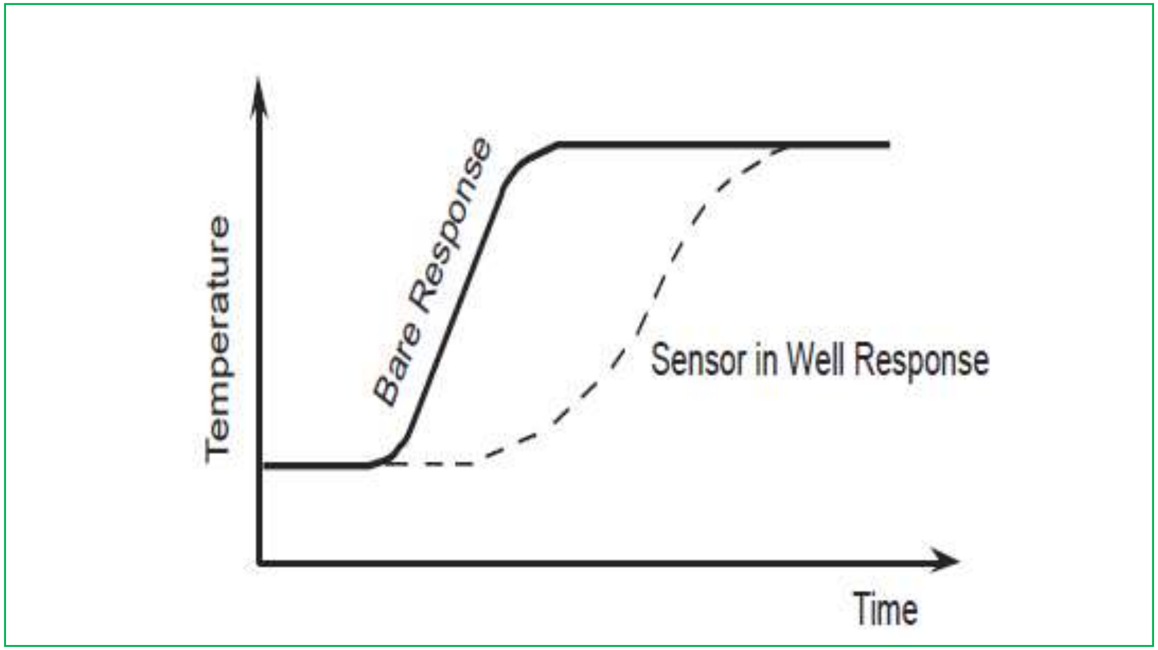
- Because the metal used for a RTD, they are much more expensive than thermocouples.
- In general, an RTD is not capable of measuring as wide a temperature range as a thermocouple.
- A power supply failure can cause erroneous readings.
- RTDs can be found in the reactor area temperature measurement and fuel channel coolant temperature.

- **Thermal Wells**
- **The process environment where temperature monitoring is required, is often not only hot, but also pressurized and possibly chemically corrosive or radioactive.**
- **A thermal well is basically a hollow metal tube with one end sealed.**
- **A drawback to thermal wells is their long response time because heat must be transferred through the well to the sensor.**



Typical Thermal Well Installation

- **An example of the temperature response for bare and thermal well installed sensors is shown in Figure.**



Response Curves of Bare and Thermal Well Installation

- Thermostats
- The thermostats directly regulate the temperature of a system by maintaining it constant or varying it over a specific range.
- The TC or RTD could be used as the temperature-sensing element of a thermostat, but generally thermostats are direct acting devices.
- The two common types of thermostats are: 1.Pressure cylinder, 2. Bimetallic strip

ELECTRONIC MEASURING INSTRUMENTS

- The instruments which are used to measure electrical quantities are called Electrical Instruments.
- Example:
- Ammeter
- Voltmeter
- Energy meter

- **Classification:**
- **Absolute Instruments:** Give the value of the quantity to be measured in terms of constants of the instrument.

  Example: Tangent Galvanometer
- **Secondary Instruments:** Determine the electrical quantity to be measured directly in terms of deflection.

- **Secondary Instruments:**
- **Indicating Instruments:** Indicate the magnitude of electrical quantity being measured instantaneously. (ammeter, voltmeter, wattmeter)
- **Integrating Instruments:** Add up the electrical quantity and measure in a given period of time. (Energy meter)

- **Recording Instruments:** Give a continuous record of the variations of the electrical quantity being measured. (ECG)

- **Types of Instruments:**
- **1. Moving Iron Instruments**
- **a) Attraction Type**
- **b) Repulsion Type**
- **2. PMMC Instruments**
- **3. Dynamometer Type Instruments**
- **4. Induction Type Instruments**

CATHODE RAY OSCILLOSCOPE

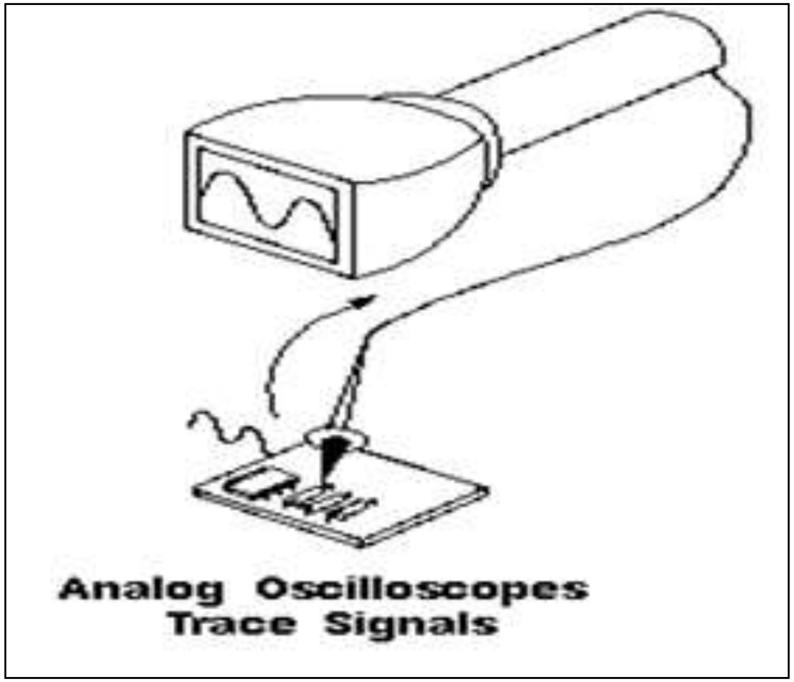
- The CRO (Cathode Ray Oscilloscope), generally referred to as the oscilloscope or simply “scope” is probably the most versatile electrical measuring instrument available.

- Measurement of electrical parameters:
  - AC or DC voltage
  - AC or DC current
  - Time
  - Phase relationship
  - Frequency

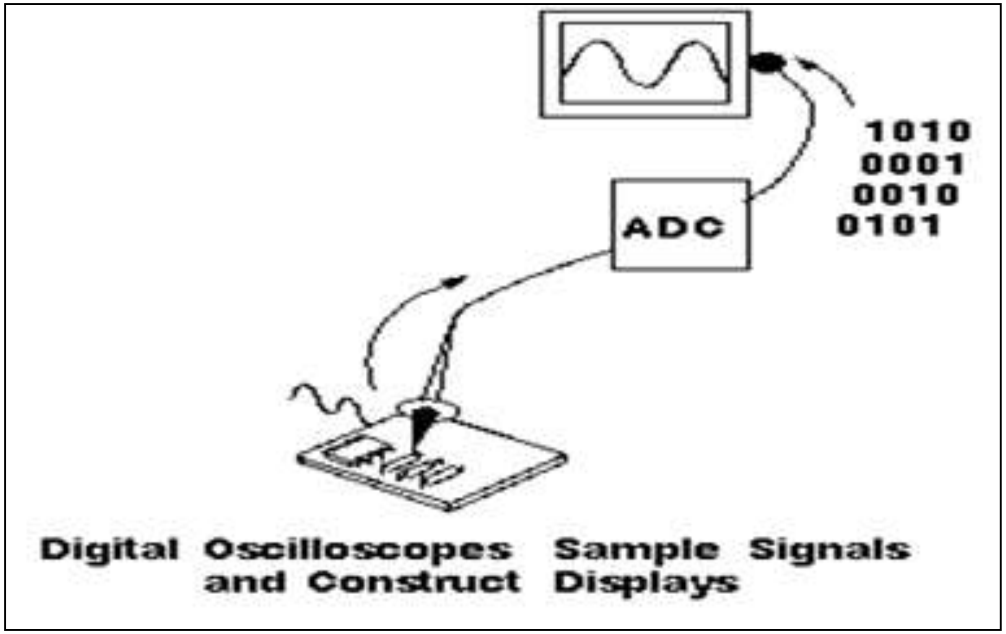
- **Oscilloscope consists of the following parts:**
  - **CRT – Cathode Ray Tube (Heart of Instrument)**
  - **Vertical amplifier**
  - **Horizontal amplifier**
  - **Sweep generator**
  - **Trigger circuit**
  - **Associated power supplier**

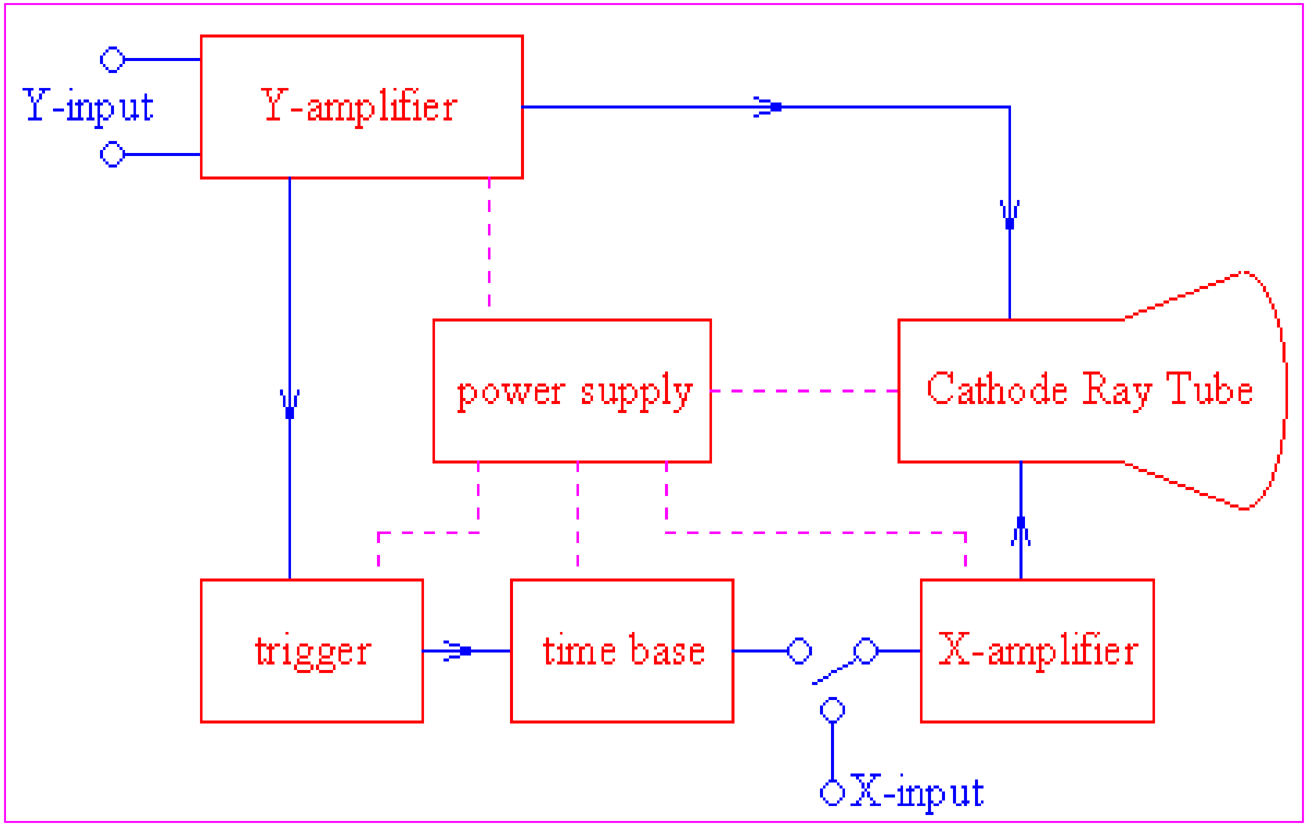
- There are 2 types of oscilloscope:
- Analog and
- Digital
- However its principle and basic characteristics still the same.

Analog Oscilloscope



Digital Oscilloscope

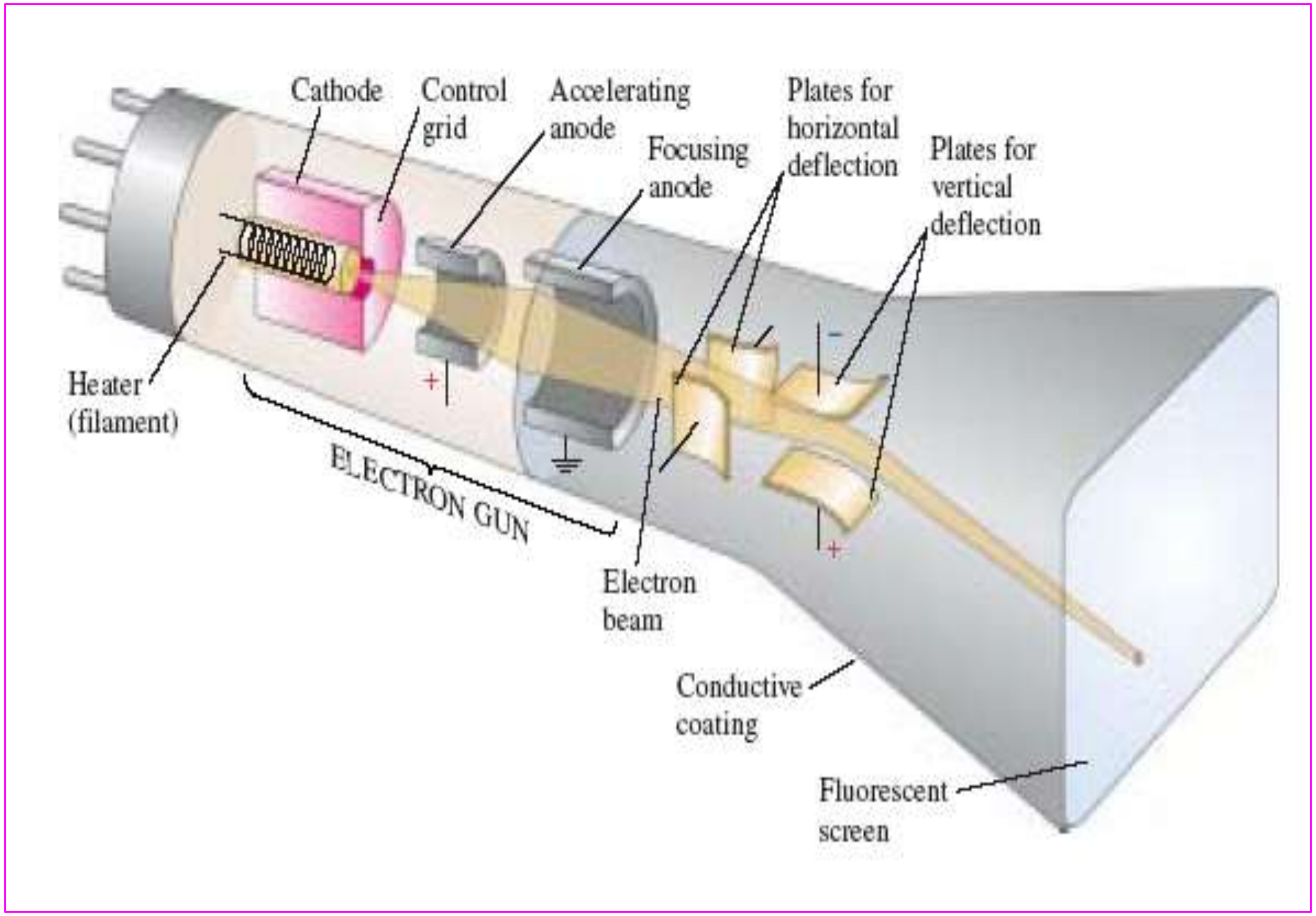




Basic Construction of CRO

- ***CRT*** is the cathode ray tube which emits electrons that strikes the phosphor screen internally to provide a visual display of signal.
- ***Vertical amplifier*** used to amplify signals in the vertical section.
- ***Delay line*** is used to delay the signal for some time in the vertical section.
- ***Time base*** used to generate saw tooth voltage required to deflect the beam in the horizontal section.

- ***Horizontal amplifier*** used to amplify saw tooth voltage before it is applied to horizontal deflection plates.
- ***Trigger circuits*** used to convert the incoming signal into trigger pulses so that input signal & the sweep frequency can be synchronized.
- ***Power supplies*** are of two types – a negative high voltage supply & a positive low voltage supply .

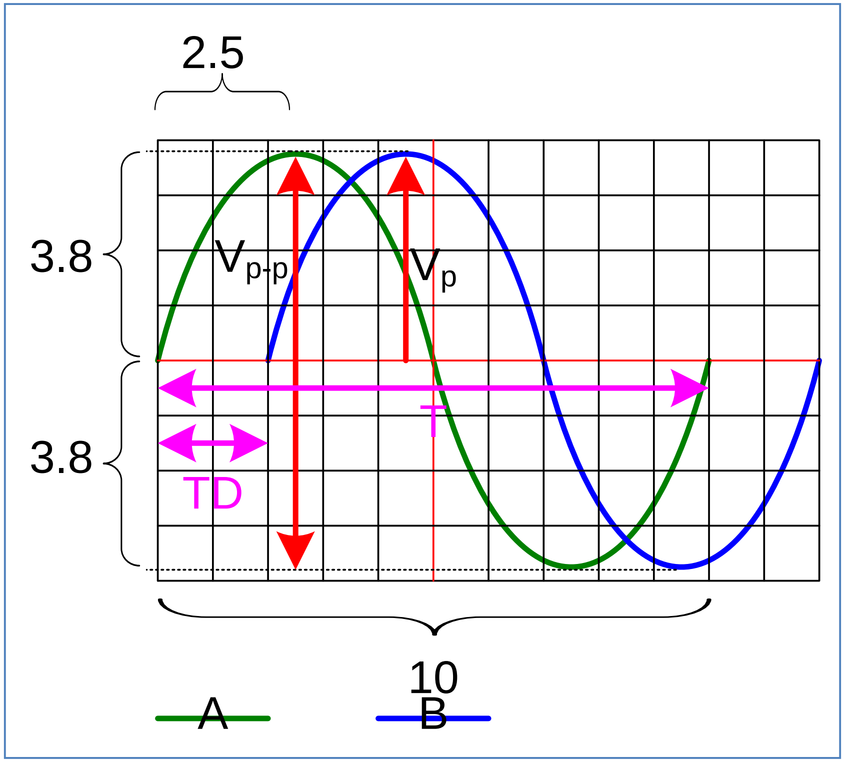


CATHODE RAY TUBE

MEASUREMENTS OF OSCILLOSCOPE

- **Voltage Measurements**
- **Period and Frequency Measurements**
- **Phase Measurements or Time Delay**

VOLTAGE MEASUREMENT



a) Voltage Peak-to-Peak

V_{p-p} = (V/Div) x No. of vert. div.

= 100 mV/div x (3.8 x 2)

= <u>0.76 V</u>

b) Voltage Peak

V_p = (V/Div) x No. of vert. div.

= 100 mV/div x (3.8)

= <u>0.38 V</u>

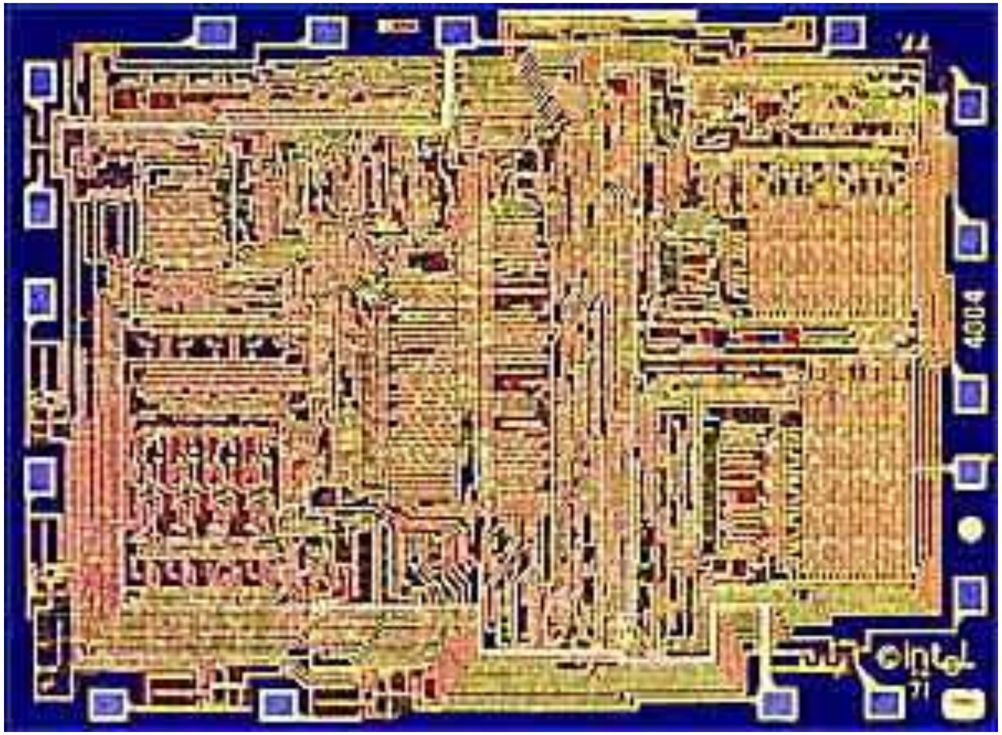
(Volt/Div : 100mV/Div, Time/Div : 0.5ms/Div)

MICROPROCESSOR

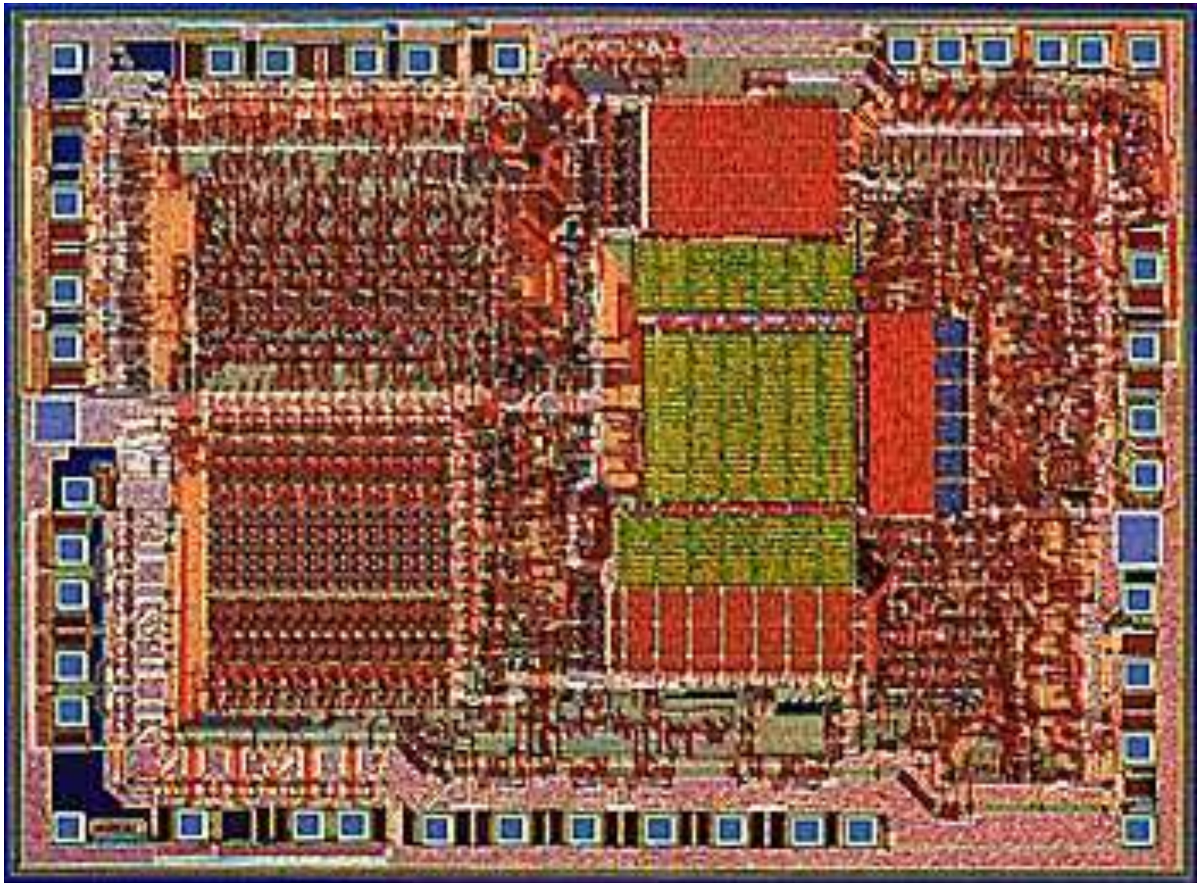
- First generation: 1971-78
  - Behind the power curve (16-bit, <50k transistors)
- Second Generation: 1979-85
  - Becoming “real” computers (32-bit , >50k transistors)

- Third Generation: 1985-89
  - Challenging the “establishment” (Reduced Instruction Set Computer/RISC, >100k transistors)
- Fourth Generation: 1990-
  - Architectural and performance leadership (64-bit, > 1M transistors, Intel/AMD translate into RISC internally)

In the beginning
(8-bit) Intel 4004



- First microprocessor-based computer
- Targeted at laboratory instrumentation
  - Mostly sold in Europe



1st Generation (16-bit) Intel 8086

- **First Microprocessor**
- **The Intel 8085 is an 8-bit microprocessor introduced by Intel in 1977.**
- **It is a simpler and less expensive microcomputer systems to be built.**

What is a Microprocessor?

- In large computers, a CPU performs these computing functions. The Microprocessor resembles a CPU exactly.
- A microprocessor incorporates most or all of the functions of a computer's central processing unit (CPU) on a single integrated circuit .

- **The microprocessor can be divided into three segments for the sake of clarity.**
- **– They are: arithmetic/logic unit (ALU),**

  - register array and

  - control unit.

Operation Types in a Microprocessor

- All of the operations of the microprocessor can be classified into one of three types:
  - Microprocessor Initiated Operations
  - Internal Operations
  - Peripheral Initiated Operations

The Design and Operation of Memory

- Memory in a microprocessor system is where information (data and instructions) is kept. It can be classified into two main types:
  - Main memory (RAM and ROM)
  - Storage memory (Disks , CD ROMs, etc.)

Applications of Microprocessor

Electronics:

- Digital clocks & Watches
- Mobile phones
- Measuring Meters

Mechanical:

- Automobiles
- Lathes
- All remote machines

Medical:

- Patient monitoring
- Most of the Medical equipments
- Data loggers

Computer:

- All computer accessories
- Laptops & Modems
- Scanners & Printers

THANK YOU

INSTRUMENTATION

- **Measurement specifications can be classified into three categories:**
- **(i) *static characteristics***
- ***(ii) dynamic characteristics and***
- ***(iii) random characteristics***

STATIC CHARACTERISTICS OF INSTRUMENTS

- **Static characteristics refer to the characteristics of the system when the input is either held constant or varying very slowly.**
- **The items that can be classified under the heading static characteristics are mainly:**

- **Accuracy**

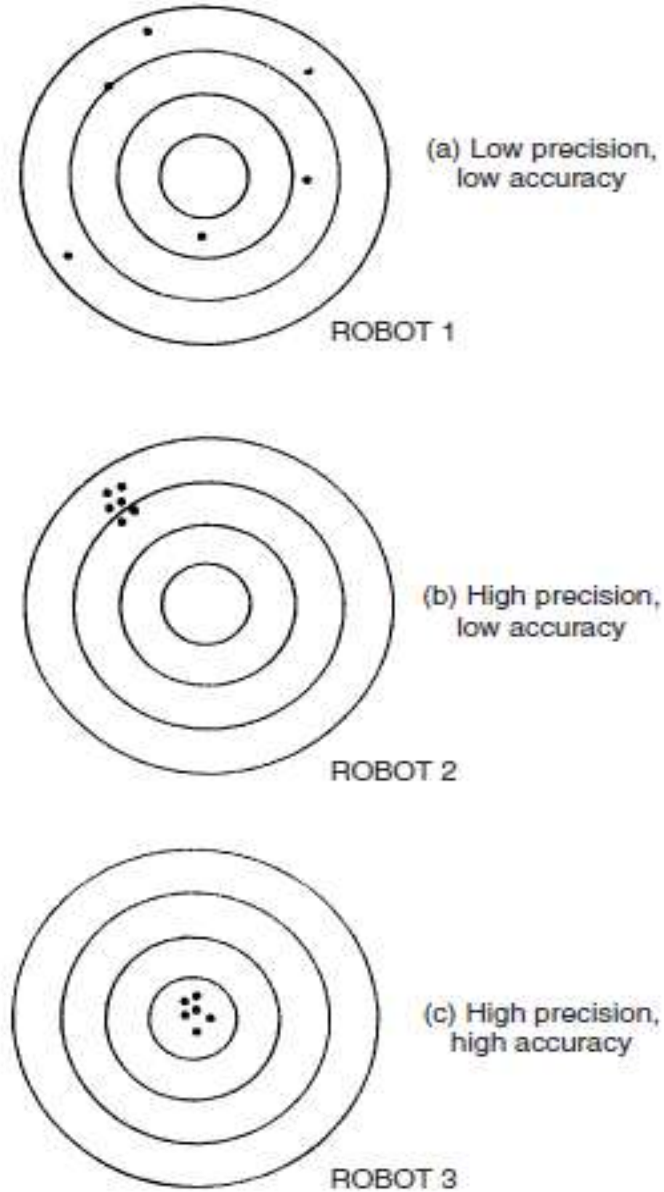
The accuracy of an instrument is a measure of how close the output reading of the instrument is to the correct value.

- **Precision**
- ***Precision is a term that describes an instrument's degree of freedom from random errors.***

- **Repeatability**
- ***Repeatability describes the closeness of output readings when the same input is applied repetitively over a short period of time*** **(with the same measurement conditions / instrument and observer / location and conditions of use throughout).**

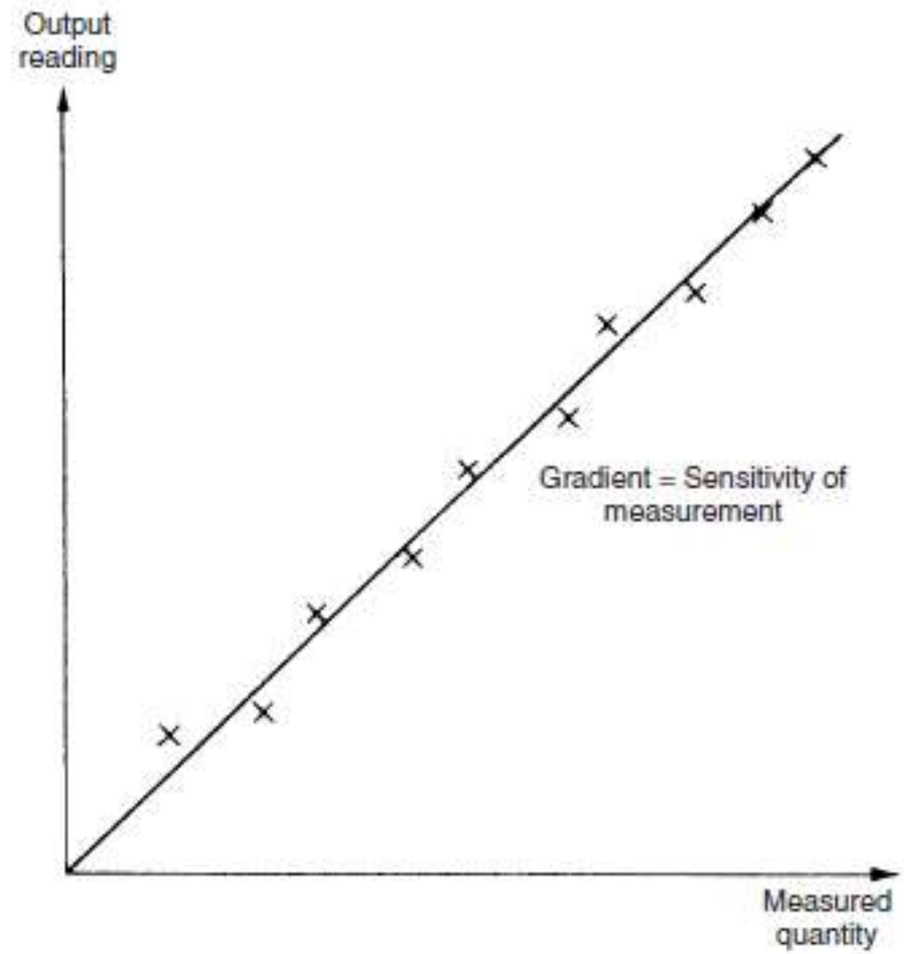
- **Reproducibility**
- ***Reproducibility describes the closeness of output readings for the same input, when there are changes in the method of measurement, observer, measuring instrument, location, conditions of use and time of measurement.***

- **Tolerance**
- ***Tolerance is a term that is closely related to accuracy and defines the maximum error that is to be expected in some value.***
- **Range or span**
- ***The range or span of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure.***



Comparison of accuracy and precision.

- **Linearity**
- It is normally desirable that the output reading of an instrument is linearly proportional to the quantity being measured.
- **Sensitivity of measurement**
- The sensitivity of measurement is a measure of the change in instrument output that occurs when the quantity being measured changes by a given amount.



Instrument output characteristic.

- **Threshold**
- **This minimum level of input is known as the *threshold* of the instrument.**

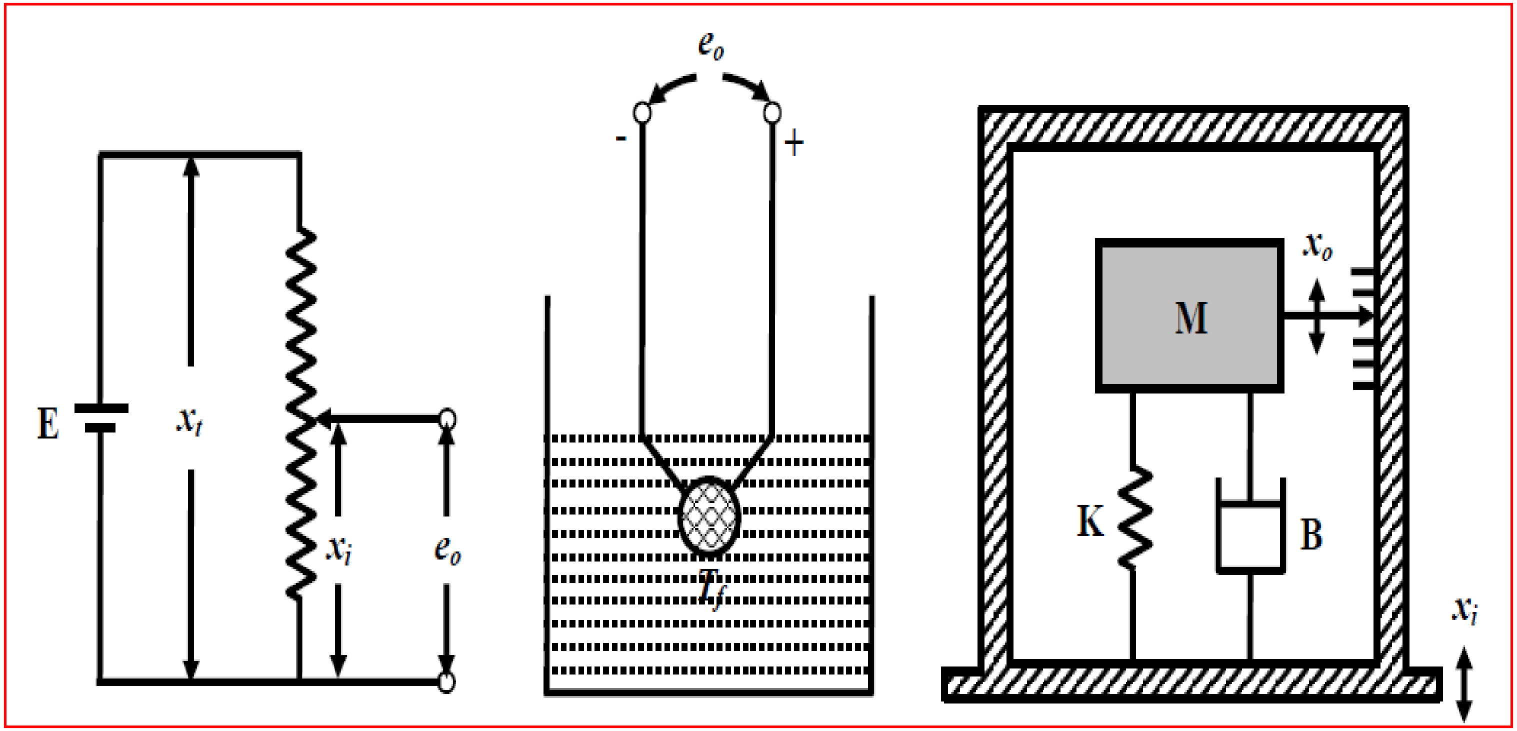
Dynamic characteristics of instruments

- The dynamic characteristics of a measuring instrument describe its behavior between the time a measured quantity changes value and the time when the instrument output attains a steady value in response.

- The dynamic performance of an instrument is normally expressed by a differential equation relating the input and output quantities.
- It is always convenient to express the input-output characteristics in form of a linear differential equation.
- Commonly sensor characteristics can be approximated as either *zero order, first order or second order dynamics.*

- **Potentiometer**
- **Displacement sensors using potentiometric principle have no energy storing elements. *So it can be termed as a zero order system.***
- **Thermocouple**
- **A bare thermocouple has a mass (*m*) *of the junction.* Hence, the bare thermocouple is a first order sensor.**

- **Seismic Sensor**
- **Seismic sensors are commonly used for vibration or acceleration measurement of foundations.**
- **It can be easily concluded that the seismic sensor is a *second order system*.**



Potentiometer Thermocouple seismic sensor

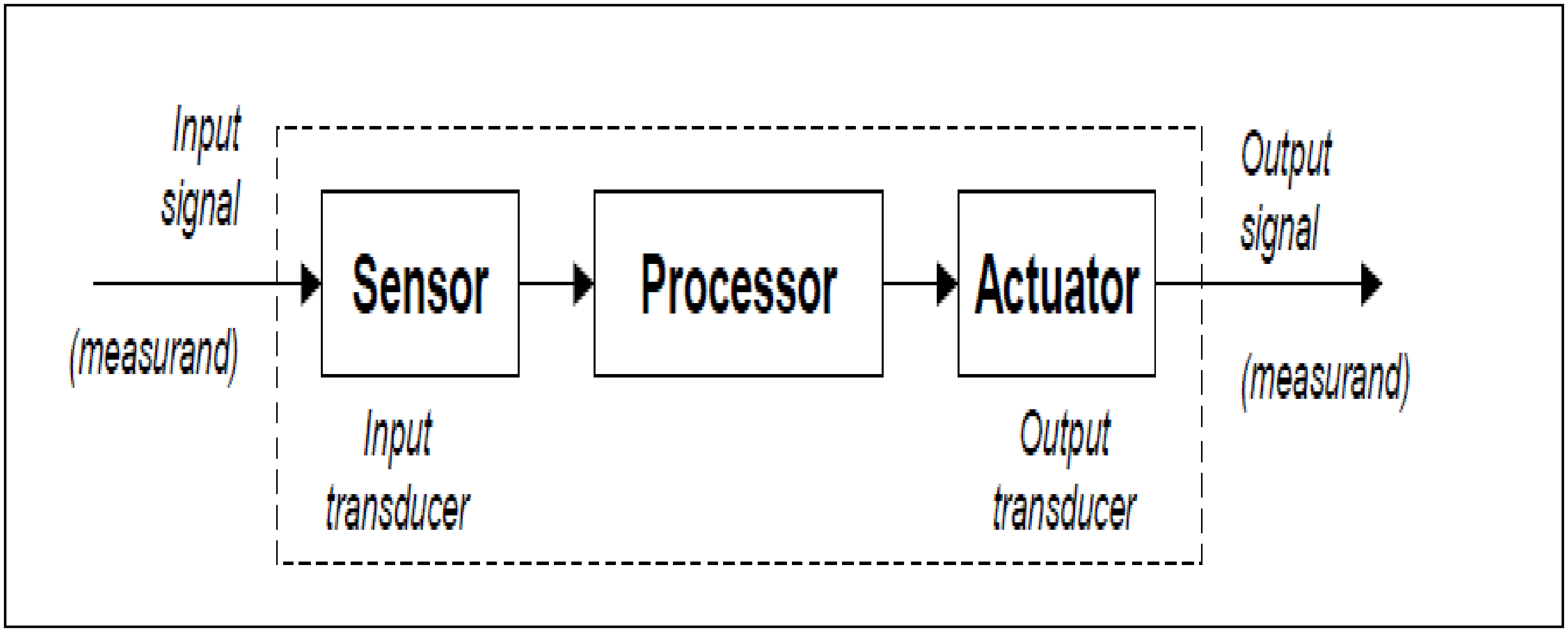
Sensor Characteristics

- **Transducers, sensors and measurements**
- **Calibration, interfering and modifying inputs**
- **Static sensor characteristics**
- **Dynamic sensor characteristics**

- **Transducer**
- **A device that converts a signal from one physical form to a corresponding signal having a different physical form.**
- **Physical form: mechanical, thermal, magnetic, electric, optical, chemical…**
- **Transducers are ENERGY CONVERTERS or MODIFIERS**

- **Sensor**
- **A device that receives and responds to a signal or stimulus.**
- **This is a broader concept that includes the extension of our perception capabilities to acquire information about physical quantities.**

- **Transducers: sensors and actuators**
- **Sensor: an input transducer (i.e., a microphone)**
- **Actuator: an output transducer (i.e., a loudspeaker)**

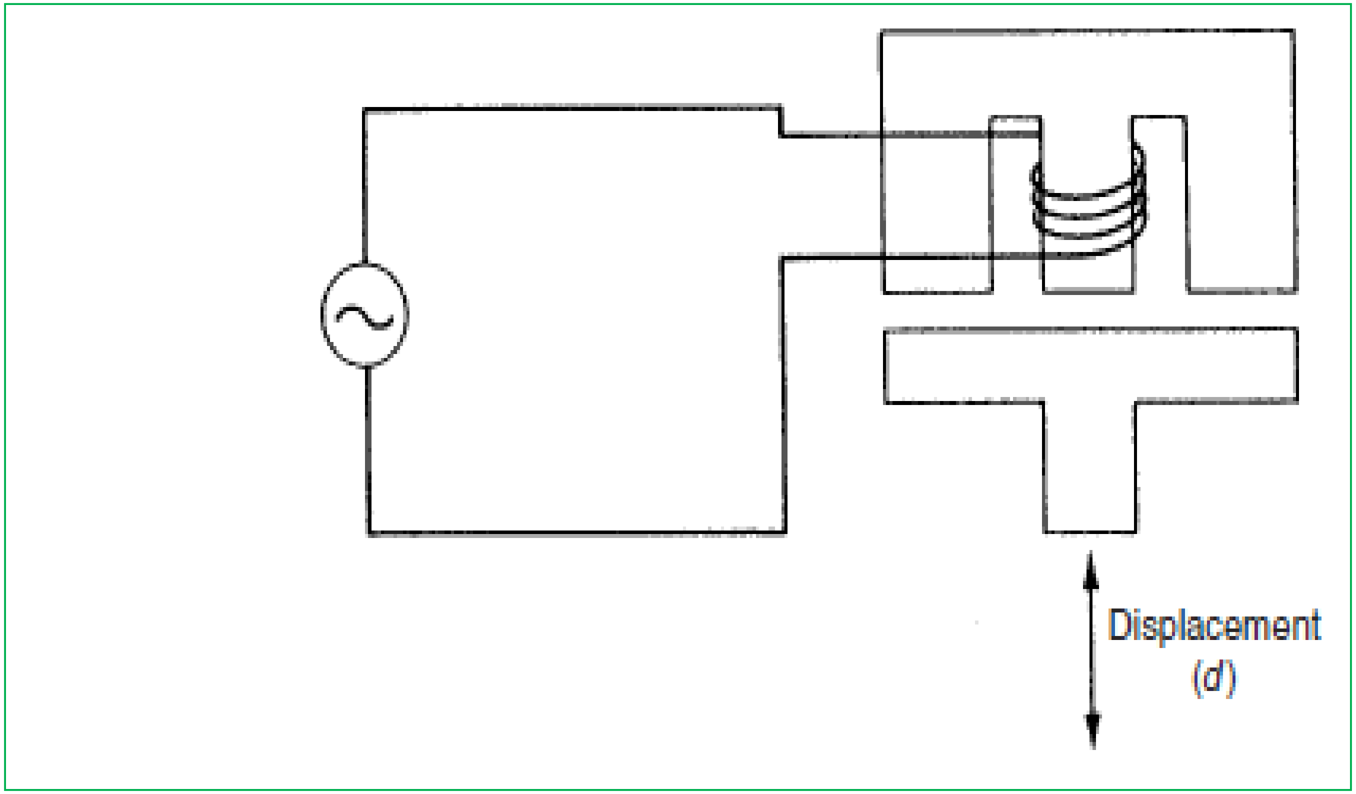

Input
signal
(measurand)
Sensor
Processor
Actuator
Output
signal
(measurand)
Input
transducer
Output
transducer

- **SENSOR TECHNOLOGIES**
- **Capacitive and resistive sensors**
- ***Capacitive sensors*** **consist of two parallel metal plates in which the dielectric between the plates is either air or some other medium.**
- **Capacitive devices are often used as displacement sensors: measuring pressure, sound or acceleration.**

- Alternatively, fixed plate capacitors can also be used as sensors, in which the capacitance value is changed by causing the measured variable to change the dielectric constant of the material between the plates in some way.
- This principle is used in devices to measure moisture content, humidity values and liquid level.

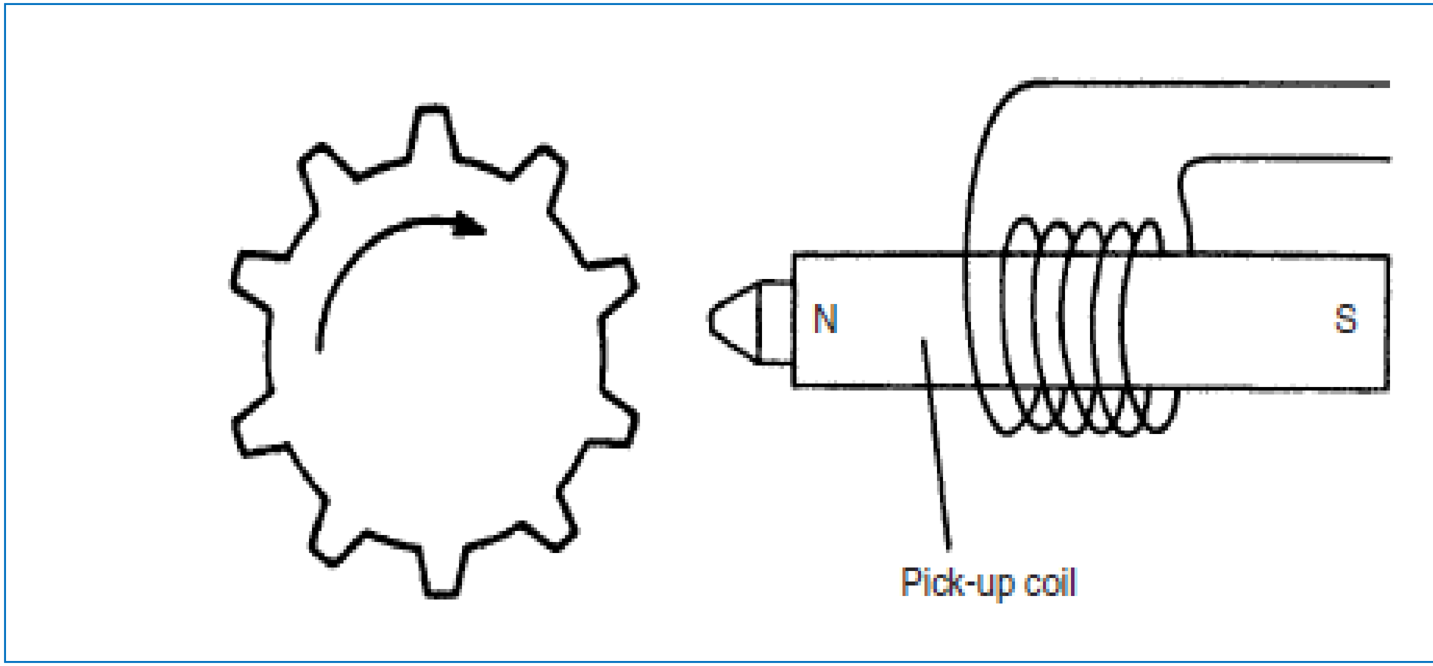
- *Resistive sensors* rely on the variation of the resistance of a material when the measured variable is applied to it.
- This principle is most commonly applied in temperature measurement using resistance thermometers or thermistors, and in displacement measurement using strain gauges or piezoresistive sensors.

- **Magnetic sensors**
- **Magnetic sensors utilize the magnetic phenomena of inductance, reluctance and eddy currents to indicate the value of the measured quantity, which is usually some form of displacement.**
- ***Inductive sensors*** **translate movement into a change in the mutual inductance between magnetically coupled parts.**



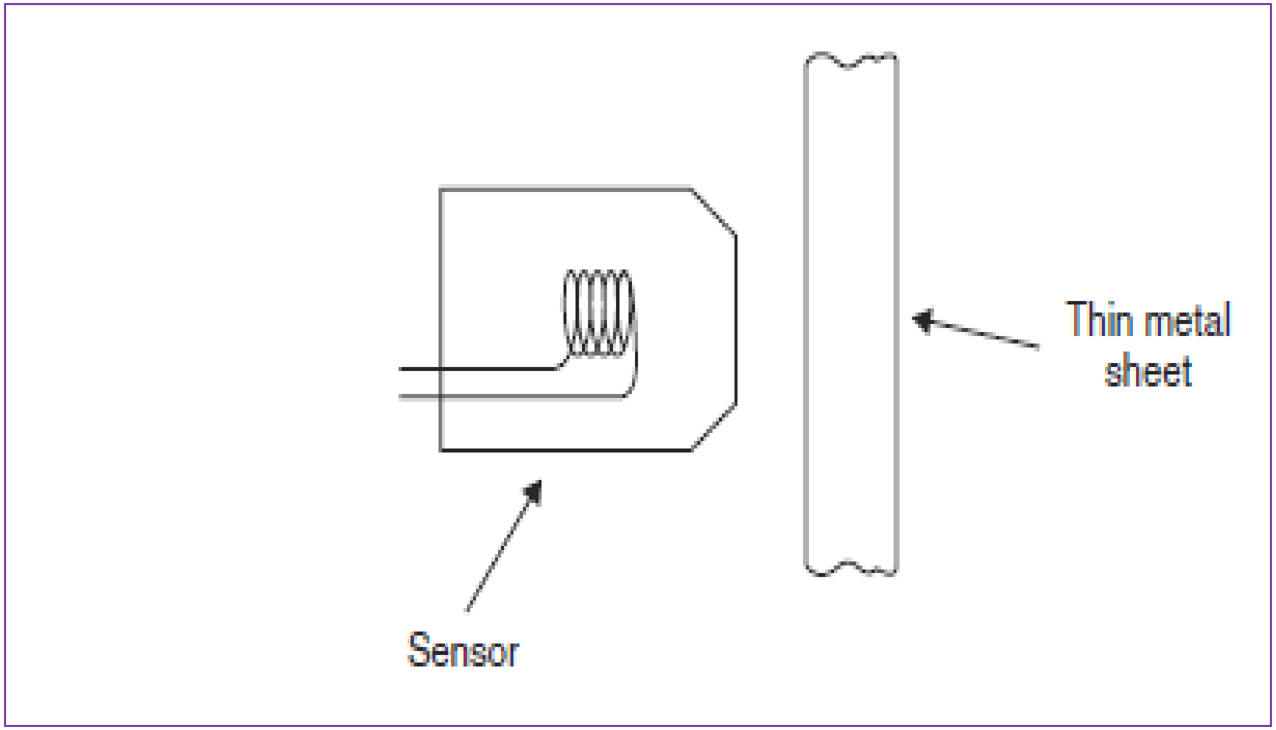
Inductive displacement sensor

- **The inductance principle is also used in differential transformers to measure translational and rotational displacements.**
- ***In variable reluctance sensors,* a coil is wound on a permanent magnet rather than on an iron core as in variable inductance sensors.**
- **Such devices are commonly used to measure rotational velocities.**



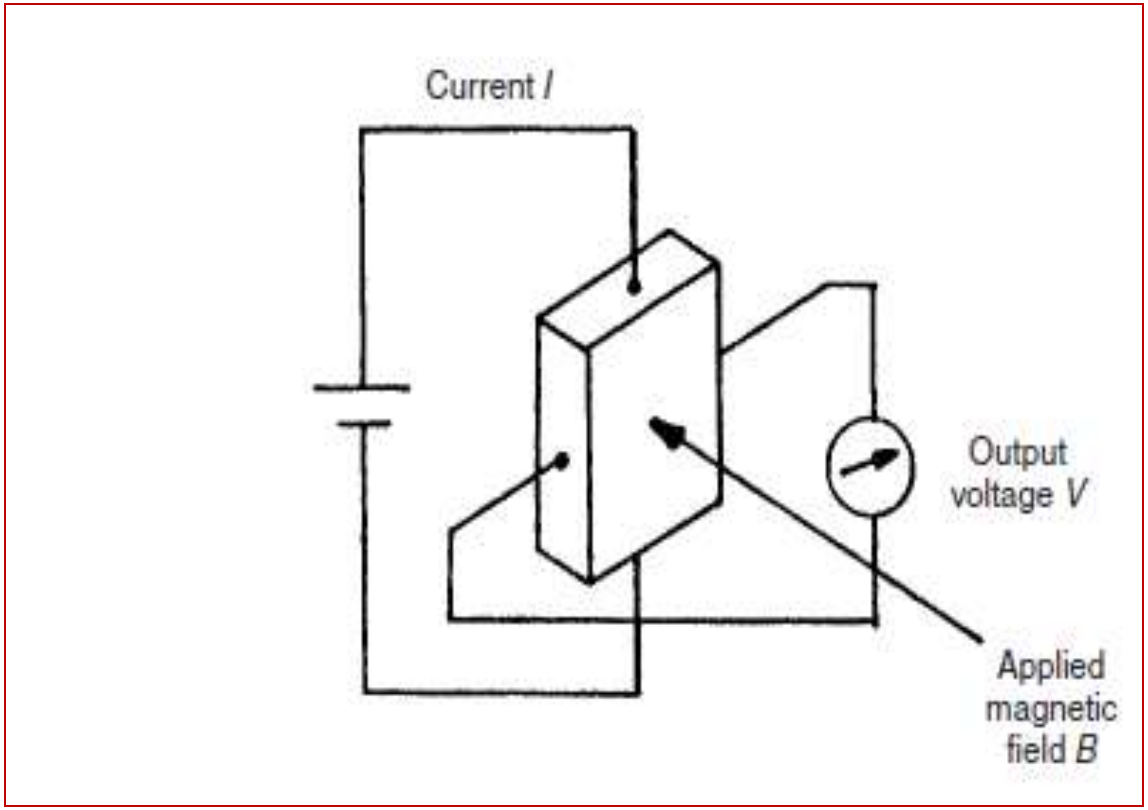
Variable reluctance sensor

- ***Eddy current sensors*** **consist of a probe containing a coil, that is excited at a high frequency, which is typically 1MHz.**
- **This is used to measure the displacement of the probe relative to a moving metal target.**



Eddy current sensor

- **Hall-effect sensors**
- **Basically, a Hall-effect sensor is a device that is used to measure the magnitude of a magnetic field.**
- **It consists of a conductor carrying a current that is aligned orthogonally with the magnetic field, as shown in Figure.**



Principles of Hall-effect sensor

- **The Hall effect is also commonly used in keyboard pushbuttons, in which a magnet is attached underneath the button.**
- **When the button is depressed, the magnet moves past a Hall-effect sensor.**

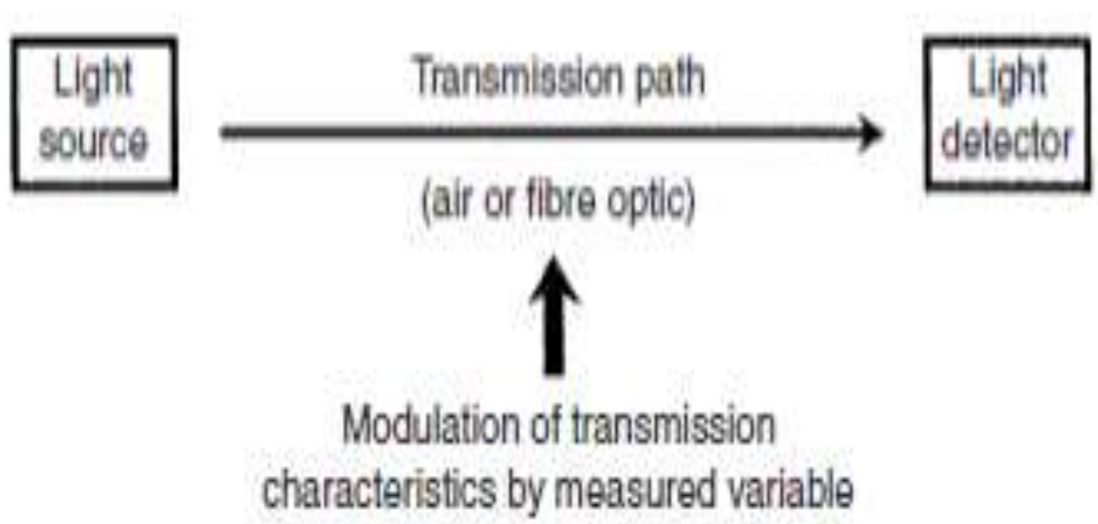
- **Piezoelectric transducers**
- **Piezoelectric transducers produce an output voltage when a force is applied to them.**
- **They are frequently used as ultrasonic receivers and also as displacement transducers, particularly as part of devices measuring acceleration, force and pressure.**

- Piezoelectric transducers are made from piezoelectric materials.
- The piezoelectric principle is invertible, and therefore distortion in a piezoelectric material can be caused by applying a voltage to it.
- This is commonly used in ultrasonic transmitters.

- **Piezoresistive sensors**
- **A piezoresistive sensor is made from semiconductor material.**
- **This is frequently used as a strain gauge, where it produces a significantly higher gauge factor than that given by metal wire or foil gauges.**
- **It is also used in semiconductor-diaphragm pressure sensors and in semiconductor accelerometers.**

- **Proper piezoelectric strain gauges, which are alternatively known as *semiconductor strain gauges,* produce most (about 90%) of their output through piezoresistive effects, and only a small proportion of the output is due to dimensional changes in the sensor.**

- **Optical sensors**
- **Optical sensors are based on the modulation of light travelling between a light source and a light detector, as shown in Figure.**
- **The transmitted light can travel along either an air path or a fiber-optic cable.**
- **Provides greater safety than electrical sensors when used in hazardous environments.**

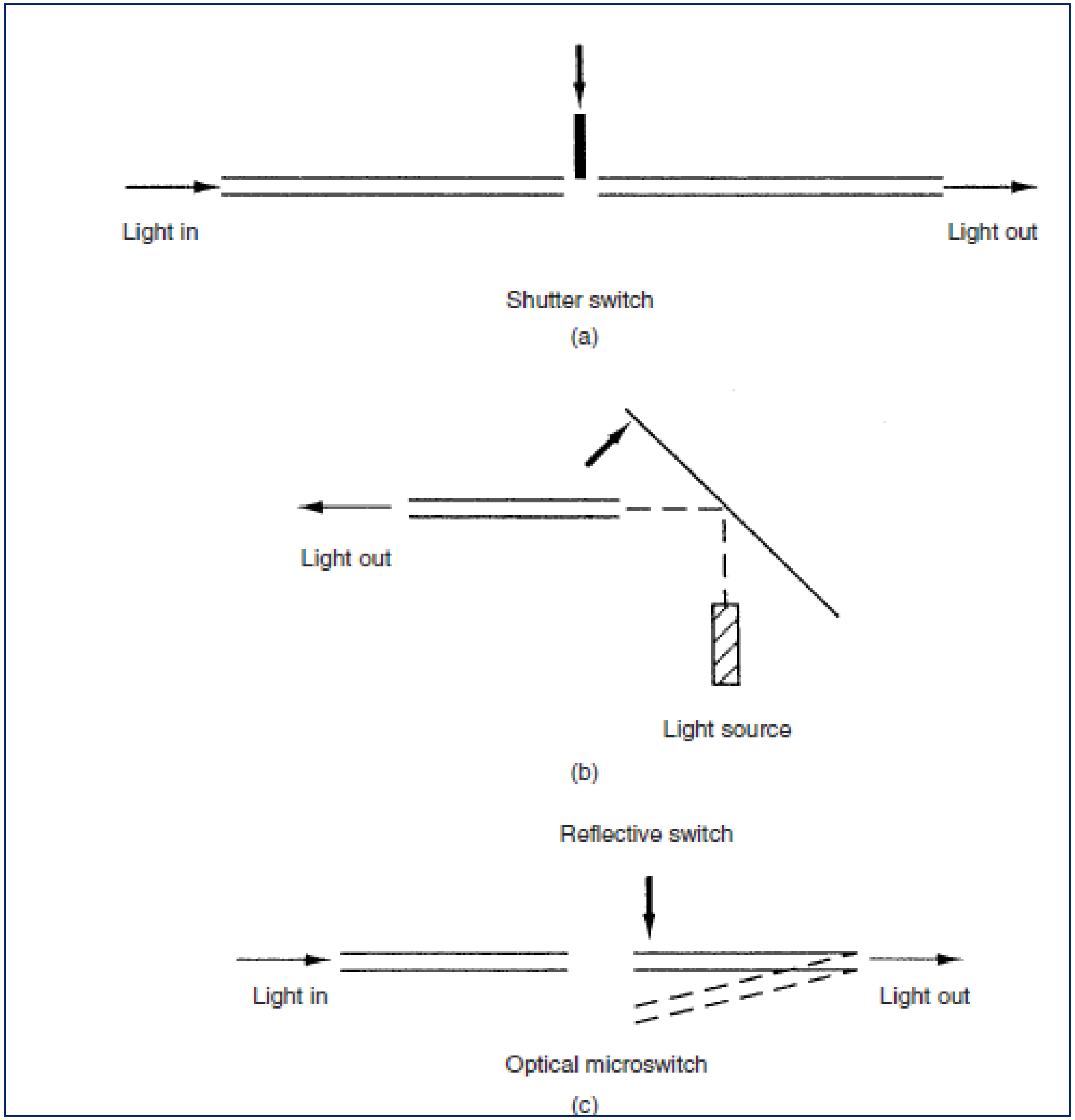


- **Operating principles of optical sensors**

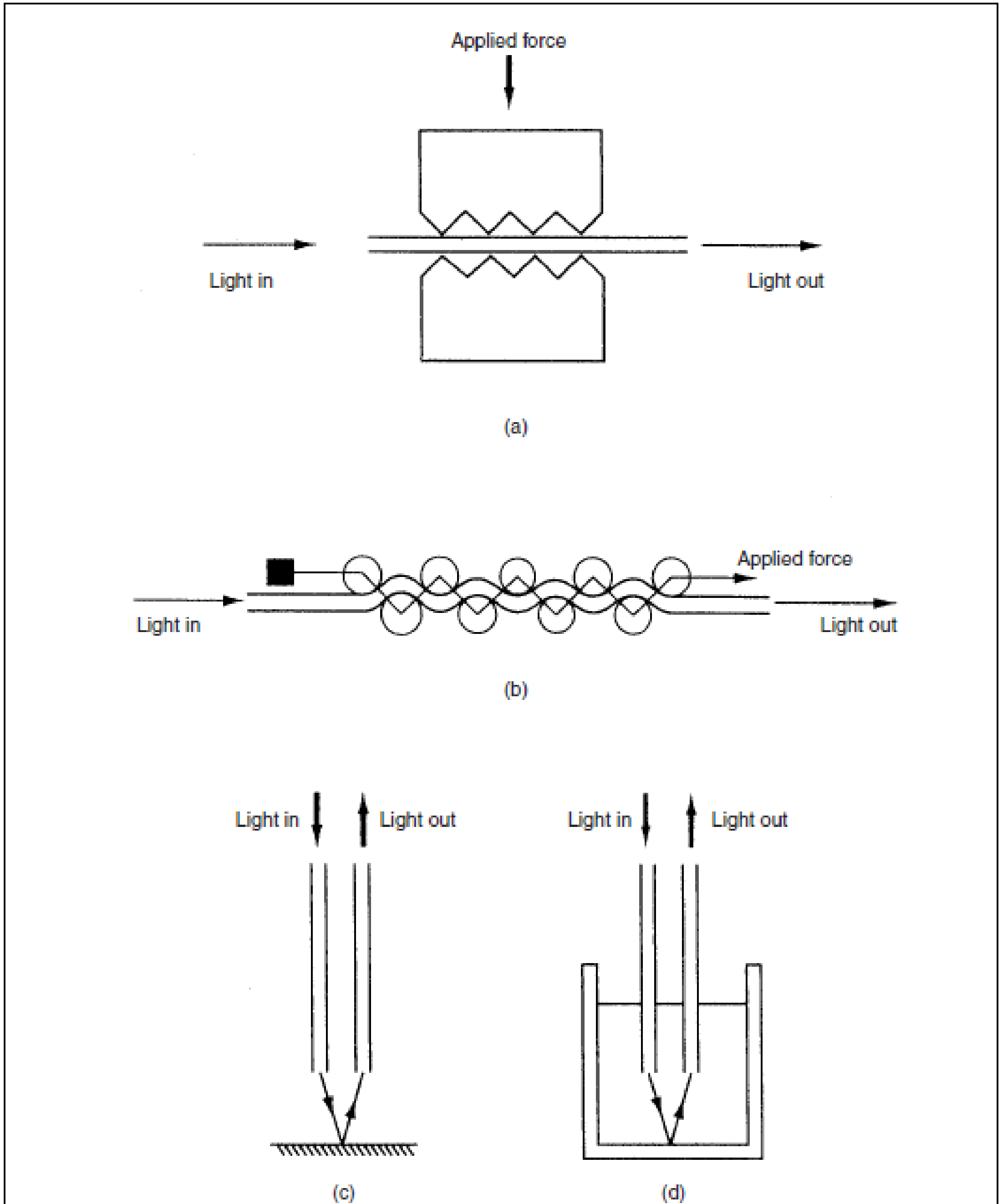
- **Intrinsic sensors**
- **Intrinsic sensors can modulate either the intensity, phase, polarization, wavelength or transit time of light.**
- **Sensors that modulate light intensity tend to use mainly multimode fibers, but only monomode cables are used to modulate other light parameters.**

- A particularly useful feature of intrinsic fiber-optic sensors is that they can, if required, provide distributed sensing over distances of up to 1 meter.
- Light intensity is the simplest parameter to manipulate in intrinsic sensors because only a simple source and detector are required.

- The various forms of switches shown in Figure, are perhaps the simplest form of these, as the light path is simply blocked and unblocked as the switch changes state.
- Modulation of the intensity of transmitted light takes place in various simple forms of proximity, displacement, pressure, pH and smoke sensors.



Intrinsic fiber-optic sensors


Applied force
Light in
Light out
(a)
Applied force
Light in
Light out
(b)
Light in
Light out
Light in
Light out
(c)
(d)

- **Ultrasonic transducers**
- **Ultrasonic devices are used in many fields of measurement, particularly for measuring fluid flow rates, liquid levels and translational displacements.**
- **Ultrasound is a band of frequencies in the range above 20 kHz, that is, above the sonic range that humans can usually hear.**

Ultrasonic sensor

- Measurement devices that use ultrasound consist of one device that transmits an ultrasound wave and another device that receives the wave.
- Changes in the measured variable are determined either by measuring the change in time taken for the ultrasound wave to travel between the transmitter and receiver.

- **Microsensors**
- Microsensors are millimeter-sized two- and three-dimensional micro machined structures that have smaller size, improved performance, better reliability and lower production costs than many alternative forms of sensor.

- **Currently, devices to measure temperature, pressure, force, acceleration, humidity, magnetic fields, radiation and chemical parameters are either in production or at advanced stages of research.**

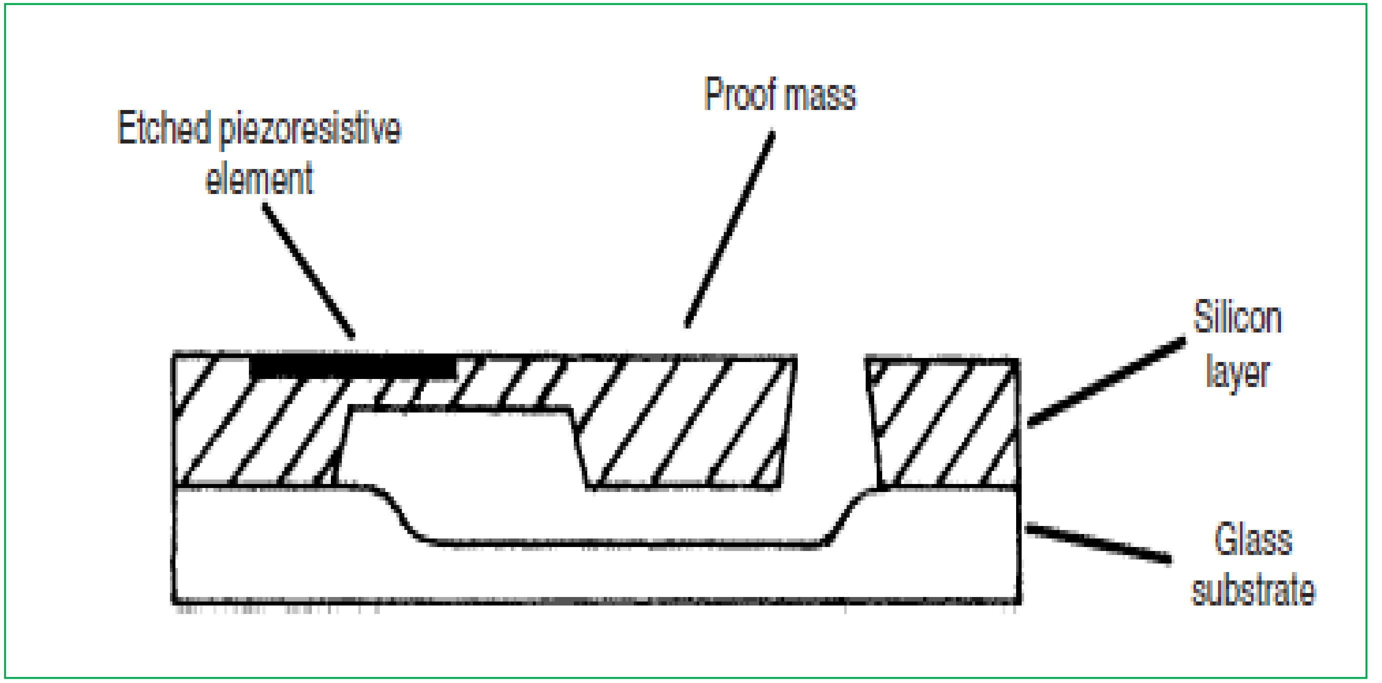
- **Microsensors are usually constructed from a silicon semiconductor material,** **but are sometimes fabricated from other materials such as metals, plastics, polymers, glasses and ceramics that are deposited on a silicon base.**
- **Silicon is an ideal material for sensor construction because of its excellent mechanical properties.**

- **Its tensile strength and Young's modulus is comparable to that of steel, whilst its density is less than that of aluminium.**
- **Sensors made from a single crystal of silicon remain elastic almost to the breaking point, and mechanical hysteresis is very small.**

- **In addition, silicon has a very low coefficient of thermal expansion and can be exposed to extremes of temperature and most gases, solvents and acids without deterioration.**
- **Micro engineering techniques are an essential enabling technology for microsensors, which are designed so that their electromechanical properties change in response to a change in the measured parameter.**

- **Many of the techniques used for integrated circuit (IC) manufacture are also used in sensor fabrication, common techniques being crystal growing and polishing, thin film deposition, ion implantation, wet and dry chemical and laser etching, and photolithography.**

- **Apart from standard IC production techniques, some special techniques are also needed in addition to produce the 3D structures that are unique to some types of microsensor.**
- **The various manufacturing techniques are used to form sensors directly in silicon crystals and films.**
- **Typical structures have forms such as thin diaphragms, cantilever beams and bridges.**

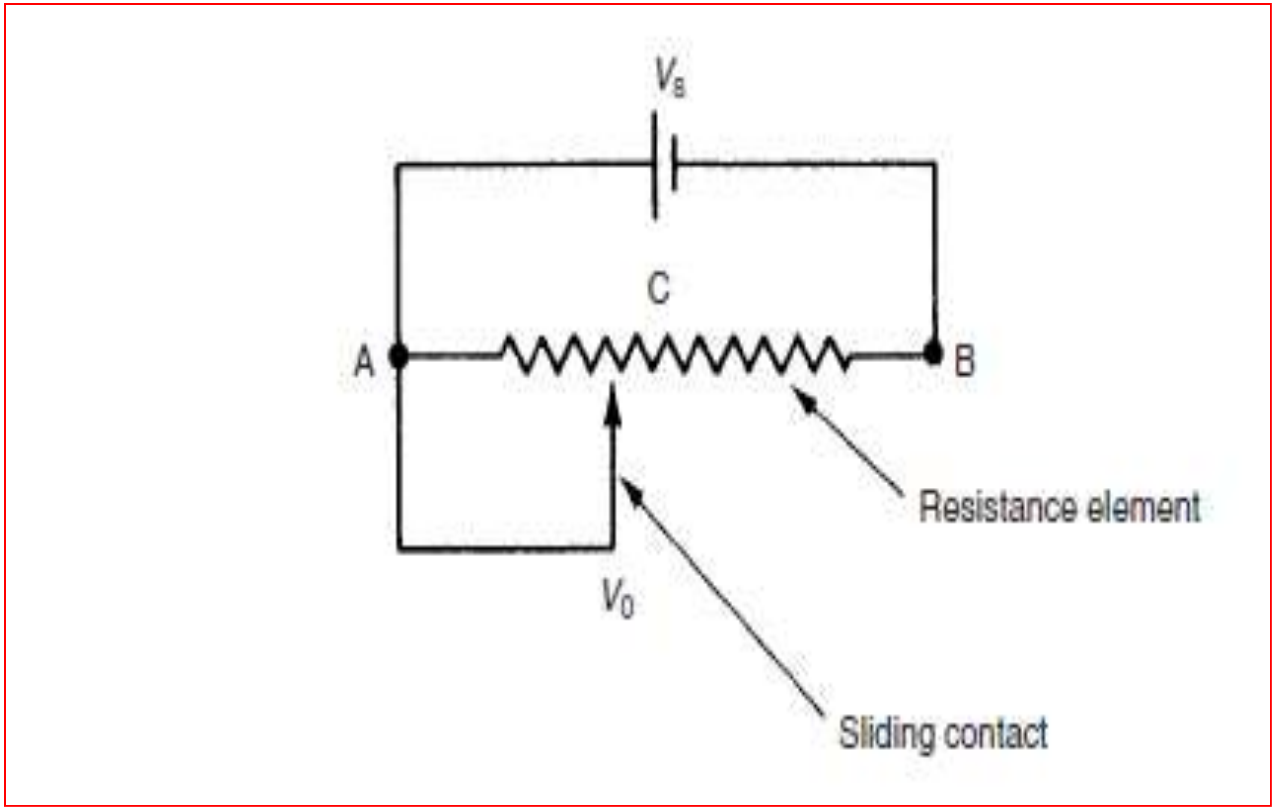


Silicon micro-accelerometer

DISPLACEMENT MEASUREMENTS

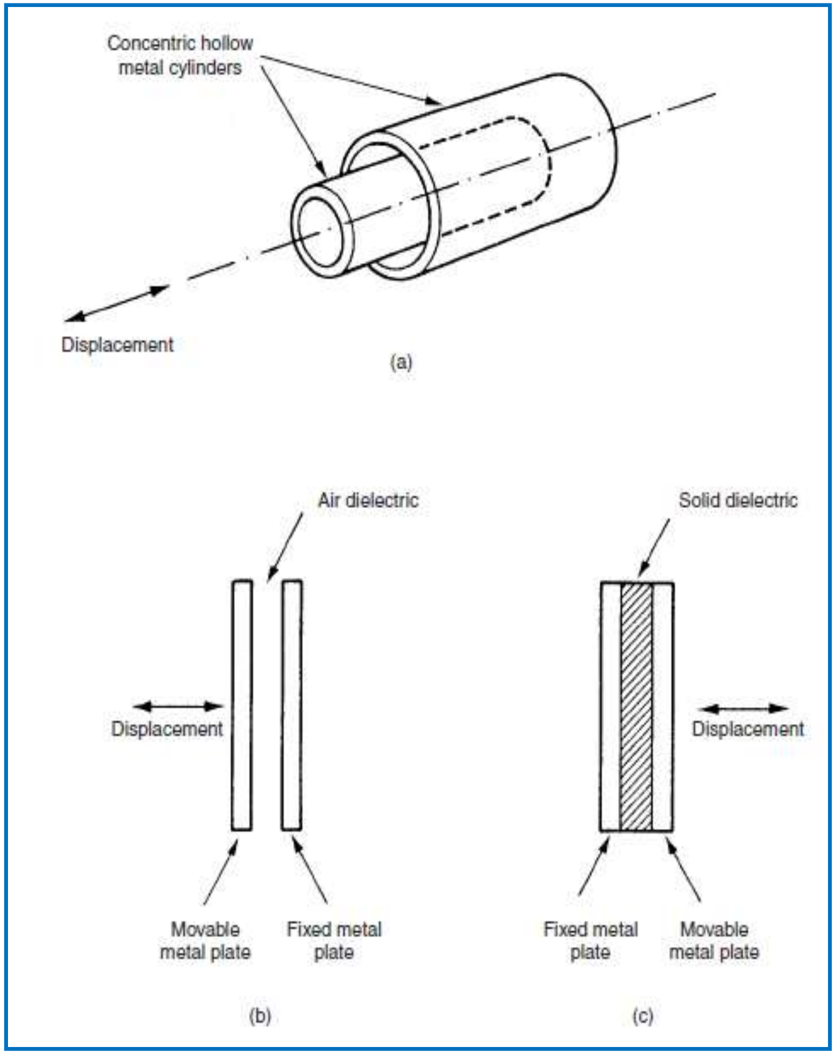
- Translational displacement transducers are instruments that measure the motion of a body in a straight line between two points.
- Other physical quantity such as pressure, force, acceleration or temperature is translated into a translational motion by the primary measurement transducer.

- The resistive potentiometer is the best-known displacement-measuring device.
- It consists of a resistance element with a movable contact as shown in Figure.
- Three different types of potentiometer exist, wire-wound, carbon-film and plastic-film. (resistance element material)



The Resistive Potentiometer

- **Variable capacitance transducers**
- **The principle of variable capacitance is used in displacement measuring transducers in various ways.**
- **The three most common forms of variable capacitance transducer are shown in Figure.**



Variable Capacitance Transducer

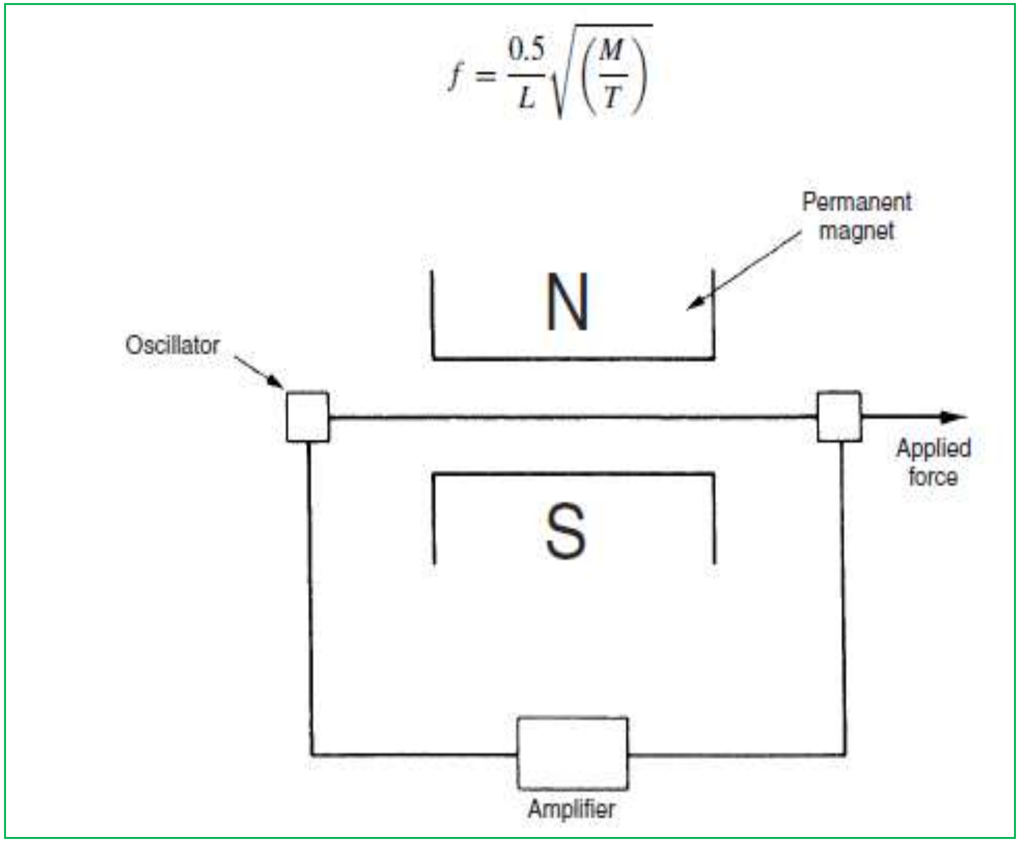
- **The major problem with variable capacitance transducers is their high impedance.**
- **Because of these difficulties, use of these devices tends to be limited to those few applications where the high accuracy and measurement resolution of the instrument are required.**

MEASUREMENT OF FORCE & TORQUE

- If a force of magnitude F, is applied to a body of mass M, the body will accelerate at a rate A, according to the equation: **F=M/A**
- The standard unit of force is the ***Newton***, force that will produce an acceleration of one meter/second^2 in the direction of the force when it is applied to a mass of one kilogram.

- Use of accelerometers
- The technique of applying a force to a known mass and measuring the acceleration produced can be carried out using any type of accelerometer.
- However, the technique can be of use in measuring some transient forces, and also for calibrating the forces produced by thrust motors in space vehicles.

- **Vibrating wire sensor**
- **This instrument, illustrated in Figure, consists of a wire that is kept vibrating at its resonant frequency by a variable-frequency oscillator.**



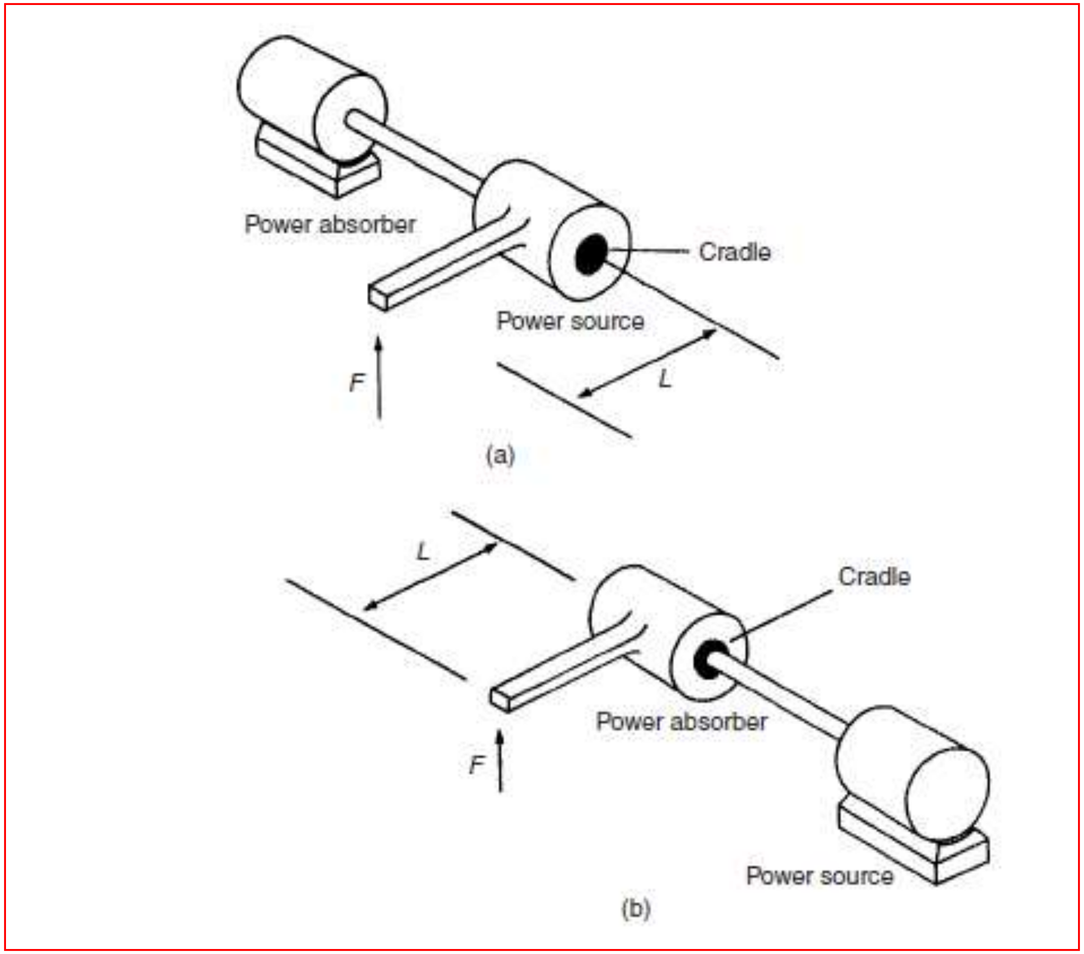
Vibrating wire sensor

- **where M is the mass per unit length of the wire, L is the length of the wire, and T is the tension due to the applied force, F.**
- **Thus, measurement of the output frequency of the oscillator allows the force applied to the wire to be calculated.**

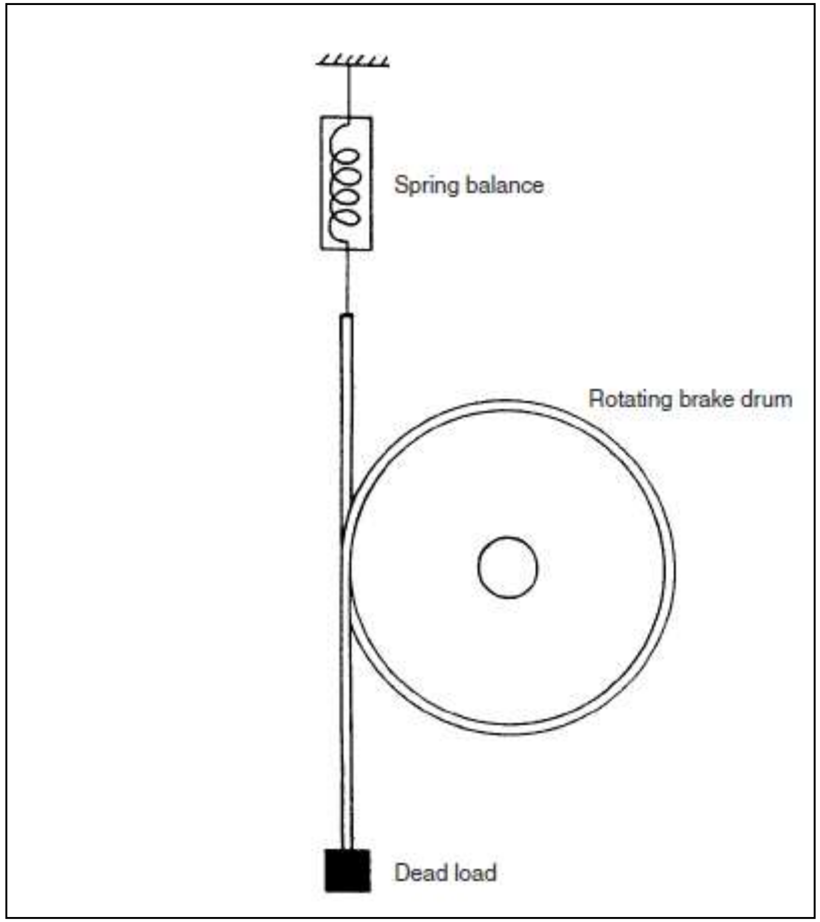
- **TORQUE MEASUREMENT**
- Measurement of applied torques is of fundamental importance in all rotating bodies to ensure that the design of the rotating element is adequate to prevent failure under shear stresses.
- Torque measurement is also a necessary part of measuring the power transmitted by rotating shafts.

- **The three traditional methods of measuring torque consist of (i) measuring the reaction force in cradled shaft bearings, (ii) the ‘Prony brake’ method and (iii) measuring the strain produced in a rotating body due to an applied torque.**
- **However, recent developments in electronics and optic fiber technology.**

- **Reaction forces in shaft bearings**
- **The magnitude of the transmitted torque can be measured by cradling either the power source or the power absorber end of the shaft in bearings, and then measuring the reaction force, F, and the arm length L, as shown in Figure.**
- **The torque is then calculated as the simple product, FL.**

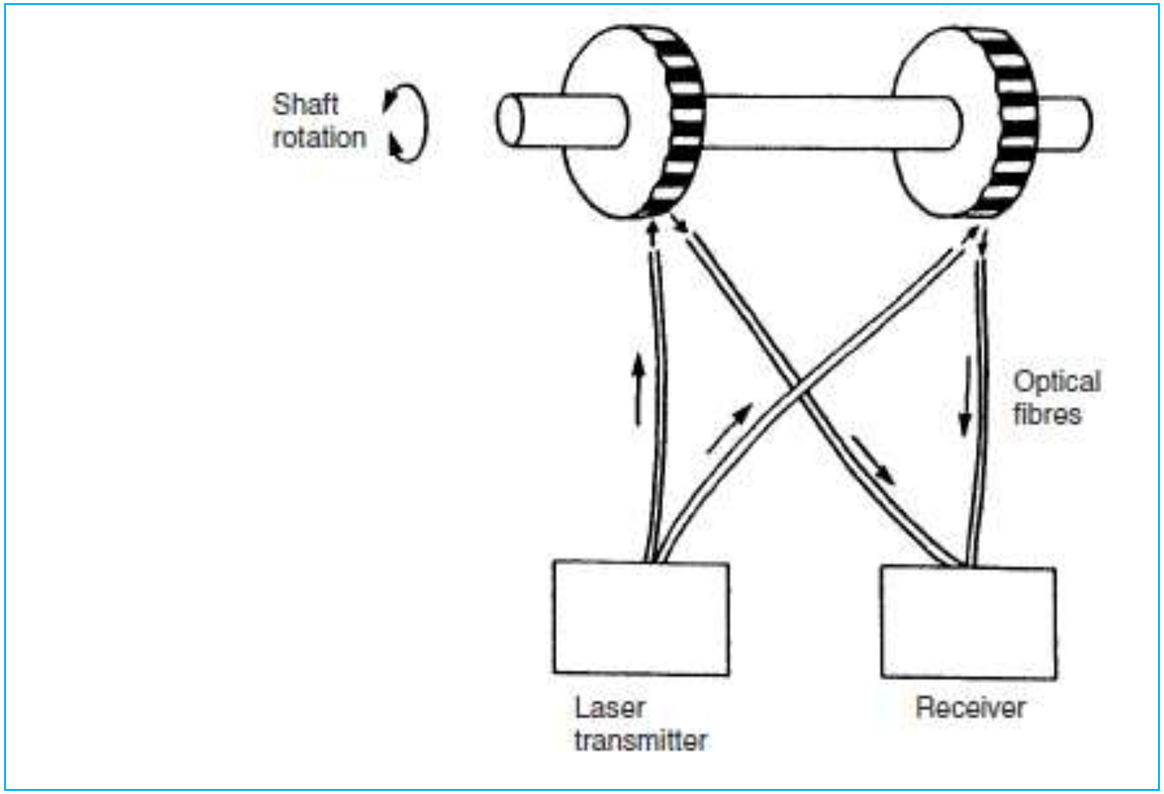


Reaction forces in shaft bearings



The Prony brake

- **Optical torque measurement**
- **Optical techniques for torque measurement have become available recently with the development of laser diodes and fiber-optic light transmission systems, shown in Figure.**
- **Two black-and-white striped wheels are mounted at either end of the rotating shaft and are in alignment when no torque is applied to the shaft.**



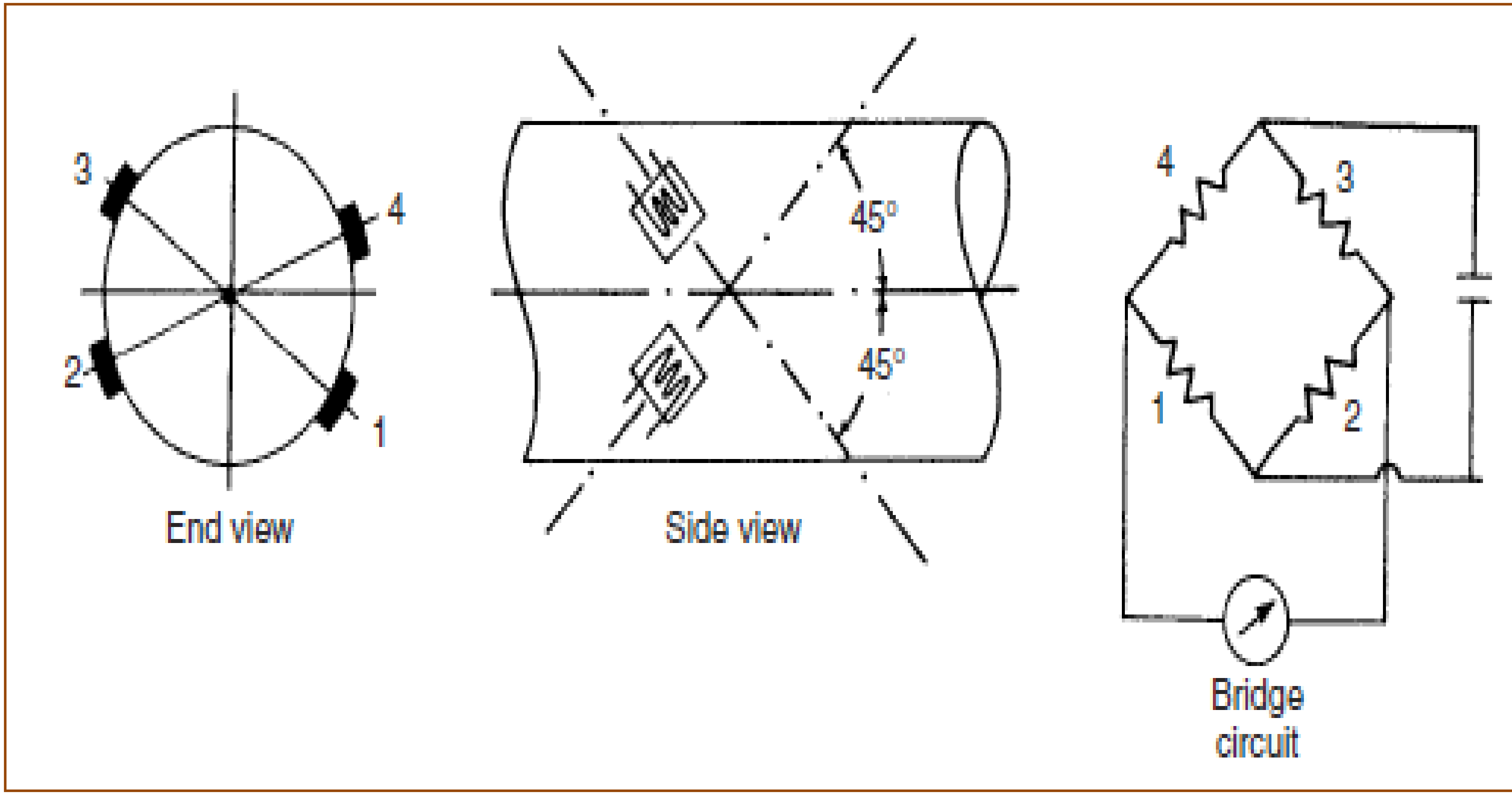
Optical torque measurement

- **Light from a laser diode light source is directed by a pair of optic-fiber cables onto the wheels.**
- **The rotation of the wheels causes pulses of reflected light and these are transmitted back to a receiver by a second pair of fiber-optic cables.**
- **Under zero torque conditions, the two pulse trains of reflected light are in phase with each other.**

- **If torque is now applied to the shaft, the reflected light is modulated.**
- **Measurement by the receiver of the phase difference between the reflected pulse trains therefore allows the magnitude of torque in the shaft to be calculated.**
- **The cost of such instruments is relatively low, and an additional advantage in many applications is their small physical size.**

- **Measurement of induced strain**
- **Measuring the strain induced in a shaft due to an applied torque has been the most common method used for torque measurement in recent years.**
- **It is a very attractive method because it does not disturb the measured system by introducing friction torques.**

- The method involves bonding four strain gauges onto the shaft as shown in Figure, where the strain gauges are arranged in a d.c. bridge circuit.
- The output from the bridge circuit is a function of the strain in the shaft and hence of the torque applied.



Position of torque-measuring strain gauges

ANALOGIES

- WHILE the analytical theory of elasticity has wide applications in the solution of engineering problems,
- it becomes very complicated and laborious when employed for investigating bodies other than those having the very simplest of geometrical shapes.

- It is because of this limitation of the mathematical theory that so much attention has been centered on photoelasticity,
- as a method for determining the difference in value between the two principal stresses at any point in a body of uniform thickness subjected to a system of loading in its own plane.

- Since this difference is equal to twice the shear stress, this analysis becomes very useful when the object to be investigated is ductile and fails according to the maximum-shear theory.
- Many of our machine parts, however, are hard and brittle and fail according to conditions predicted by the maximum-normal stress theory.

- Therefore, paralleling the development of photoelasticity there have been attempts to supplement its results by other methods, making it possible to obtain the sum of the principal stresses.
- The methods proposed have not been sufficiently accurate or simple in their execution to warrant their widespread adoption by engineers.

Because of this, scientists and engineers concerned with the design of such systems have, with increasing frequency, made use of analogies.

- **Membrane Analogy**
- **Sand-Heap Analogy**
- **Electrical Analogy**

Membrane analogy:

- The elastic membrane analogy, also known as the soap-film analogy, was first published by aerodynamicist Ludwig Prandtl in 1903.
- It describes the stress distribution on a long bar in torsion.
- The cross section of the bar is constant along its length, and need not be circular.

- **The differential equation that governs the stress distribution on the bar in torsion is of the same form as the equation governing the shape of a membrane under differential pressure.**
- **Therefore, in order to discover the stress distribution on the bar, all one has to do is cut the shape of the cross section out of a piece of wood,**

- cover it with a soap film, and apply a differential pressure across it.
- Then the slope of the soap film at any area of the cross section is directly proportional to the stress in the bar at the same point on its cross section.

- **While the membrane analogy allows the stress distribution on any cross section to be determined experimentally,**
- **it also allows the stress distribution on thin-walled, open cross sections to be determined by the same theoretical approach that describes the behavior of rectangular sections.**

- Using the membrane analogy, any thin-walled cross section can be "stretched out" into a rectangle without affecting the stress distribution under torsion.
- The maximum shear stress, therefore, occurs at the edge of the midpoint of the stretched cross section, and is equal to , where T is the torque applied, b is the length of the stretched cross section, and t is the thickness of the cross section.

- It can be shown that the differential equation for the deflection surface of a homogeneous membrane, subjected to uniform lateral pressure and with uniform surface tension and with the same outline as that of the cross section of a bar under torsion, has the same form as that governing the stress distribution over the cross section of a bar under torsion.

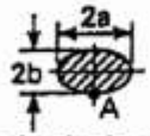
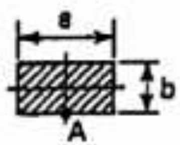
Prandt's Stress Function

$\tau_{zx} = \frac{\partial \emptyset}{\partial y}$ $\quad$ $\tau_{zy} = \frac{\partial \emptyset}{\partial x}$ $\quad$ and $\quad \frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = -2G\theta$

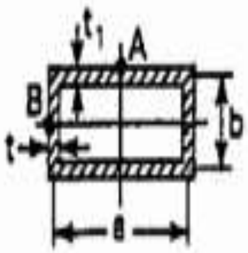
Bcs $\quad \emptyset = 0 \quad$ on the boundaries

$$T = 2\iint \emptyset dxdy = 2(Area\ under\ \emptyset\ curve)$$

$\tau_{max} = \frac{Tr}{J}$ $\quad$ twist ➔ $\emptyset = \theta L = \frac{TL}{JG}$

Cross section	*Maximum shearing stress*	*Angle of twist per unit length*
 **For circular bar: $a = b$**	$\tau_A = \dfrac{2T}{\pi a b^2}$	$\theta = \dfrac{(a^2 + b^2)T}{\pi a^3 b^3 G}$
 **Equilateral triangle**	$\tau_A = \dfrac{20T}{a^3}$	$\theta = \dfrac{46.2T}{a^4 G}$
	$\tau_A = \dfrac{T}{\alpha a b^2}$	$\theta = \dfrac{T}{\beta a b^3 G}$

a/b	β	α
1.0	0.141	0.208
1.5	0.196	0.231
2.0	0.229	0.246
2.5	0.249	0.256
3.0	0.263	0.267
4.0	0.281	0.282
5.0	0.291	0.292
10.0	0.312	0.312
∞	0.333	0.333

	$\tau_A = \frac{T}{2abt_1}$ $\tau_B = \frac{T}{2abt}$	$\theta = \frac{(at + bt_1)T}{2tt_1a^2b^2G}$
 For circular tube: $a = b$	$\tau_A = \frac{T}{2\pi abt}$	$\theta = \frac{\sqrt{2(a^2 + b^2)}T}{4\pi a^2b^2tG}$
 Hexagon	$\tau_A = \frac{5.7T}{a^3}$	$\theta = \frac{8.8T}{a^4G}$

- **Several general conclusions about torsional stiffness of a prismatic member can be drawn using the membrane analogy:**
- **(1) A bar which is long and narrow in cross section will be less stiff than a square bar of the same cross sectional area.**

- (2) Any long narrow section of U, L or C shape has approximately the same torsional stiffness as a rectangular bar of the same thickness and total length of section.
- This is known as Bach'S approximation.
- The maximum shearing stresses will occur where the largest inscribed circle touches the edge of the section in question.
- This is the middle of the longest side for a rectangular section.

Other applications:

- Prandtl's stretched-membrane concept was used extensively in the field of electron tube ("vacuum tube") design (1930's to 1960's) to model the trajectory of electrons within a device.
- The model is constructed by uniformly stretching a thin rubber sheet over a frame, and deforming the sheet upwards with physical models of electrodes, impressed into the sheet from below.

- The entire assembly is tilted, and steel balls (as electron analogs) rolled down the assembly and the trajectories noted.
- The curved surface surrounding the "electrodes" represents the complex increase in field strength, the electron-analog approaches the "electrode"; the upward distortion in the sheet is a close analogy to field strength.

Sand-Heap Analogy

- **By arranging an elasto-plastic stress-strain curve the theory of plasticity can be *made to compute the stresses in those areas of the cross* section where yielding has occurred.**
- **If for pure torsion on a prismatic member the shearing stress reaches the yield point then the components τ_{xz} & τ_{yz} must satisfy the condition:**

- $\tau_{xz}^2 = \tau_{yz}^2$ constant $= \tau^2$
- These components must also satisfy the equilibrium equations,
- $(\delta\tau_{xz} / \delta x) + (\delta \tau_{yz} / \delta y) = 0$
- Including plastic stress function under vector notation,
- $-\tau_{yz}\,dx + \tau_{xz}\,dy =$

$$(\delta F/\delta x)dx + (\delta F/\delta y)dy = 0$$

- **Thus, the plastic stress function is a constant along the boundary.**
- **This constant is usually set equal to zero since its value does not affect the value of the stresses.**
- **Since the slope of the stress function is constant the volume enclosed by ‘F’ can be likened to the shape of the sand heap obtained,**

- if sand is piled on a flat horizontal plate of the same shape as the cross section of the prism subjected to torsion; hence the sand-heap analogy.
- For a fully-plastic condition across the section several general conclusions about the torsional stiffness of a prismatic member can be made using the sand-heap analogy:

- The shearing stress across the section is constant.
- The total twisting moment applied to the bar is directly proportional to the volume of the sand heap.
- The value of the stress function is independent of the angle of twist.
- A bar which is long and narrow in cross section will not be as strong as a square bar of the same cross sectional area.

- **(5) Any long narrow section of U, T, L or C shape has approximately the same plastic resistance torque as a rectangular bar of the thickness and total length.**
- **From the sand-heap analogy, the surface of stress function; F for a rectangular section is:**

- $T_u = \frac{1}{2} b^2 (d - 1/3\ b)\ \tau_{max}$
- where , T_u = Plastic torque
- τ_{max} = maximum shearing stress for the material
- d = depth of the cross section
- b = width of the cross section

ELECTRICAL ANALOGIES

- **Analogs have long been used by men, maps perhaps represent one of the earliest useful forms.**
- **Kirchhoff in 1845 utilized the analogy between Current flow in a plate and flux distribution between charged parallel-line conductors in a uniform medium to determine the capacitance between certain configurations of such conductors.**

- The instrument constructed was of the "fast time" type in which a particular problem is solved repetitively at a speed much higher than that used for the thermal prototype.
- The ratio of the speed of the electrical operation to the speed of the thermal process was of the order of 106•

- This requires cathode-ray presentation for read-out of the repetitively solved problem.
- In order to reduce the cost and time involved in its construction, commercial electronic instruments were used wherever it seemed practical.
- This has been divided into three interrelated functional units: (a) The signal generator (b) the electrical model and (c) the measuring circuits.

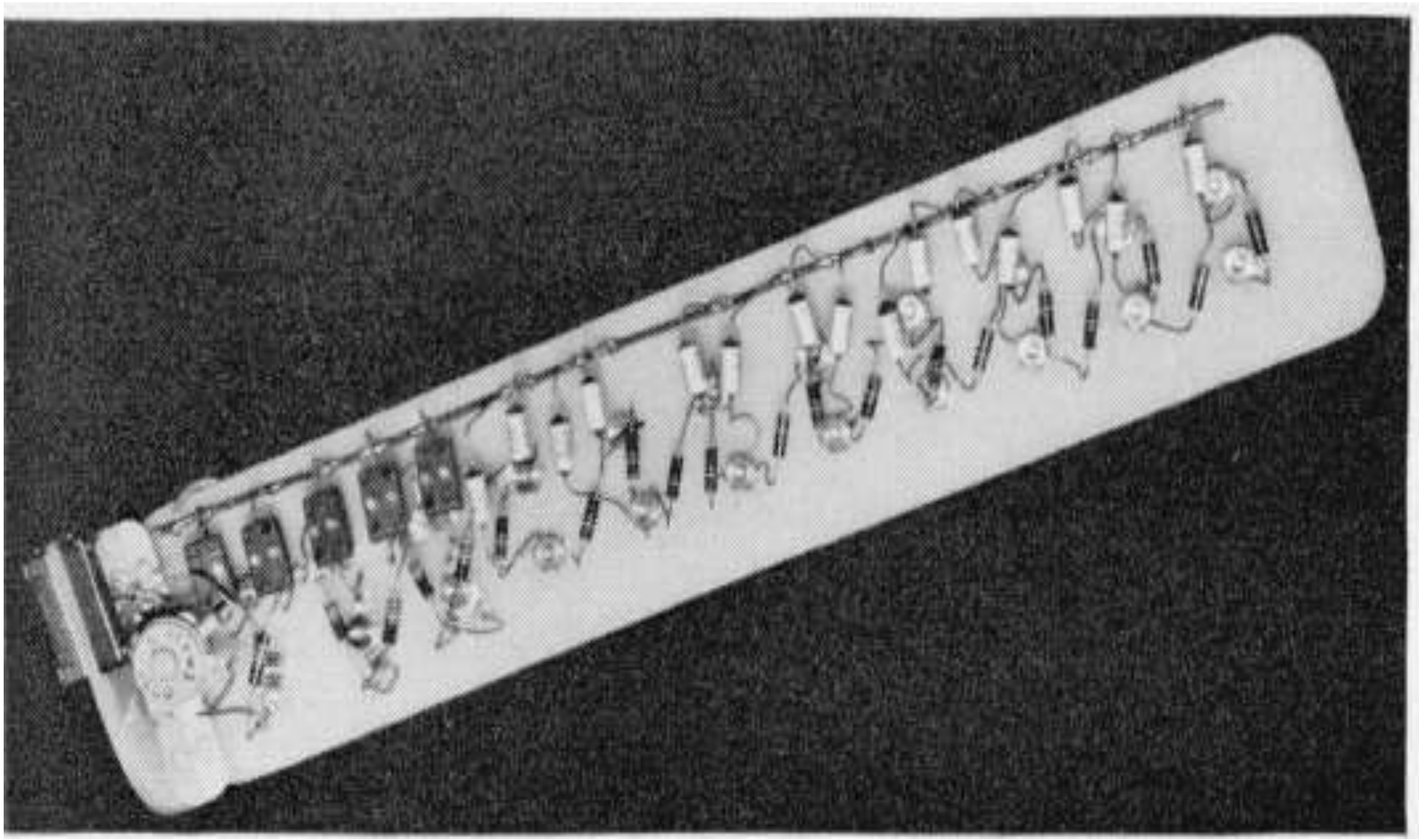
- **The signal generator unit selected was of the photoformer type.**
- **This type was chosen because of the great flexibility of waveforms possible with such a system and the ease with which they could be interchanged.**
- **A cathode-ray tube is arranged with an opaque mask covering a portion of its face.**

- **A phototube with appropriate amplifier is arranged to view the face of the cathode-ray tube.**
- **The amplified output of the phototube is applied to the vertical deflection plates of the cathode-ray tube in such a way that when the spot is "seen" by the phototube it is deflected downwards and forced to be partially hidden by the mask.**

- **The application of a relaxation-type timing sweep to the horizontal plates of the cathode-ray tube results in the spot following the mask outline on the cathode-ray tube screen .**
- **Since for given operating conditions the position of the spot on the screen bears a fixed relationship to the voltage applied to the deflection plates, a voltage-time signal may be taken from them which is controlled by the shape of the mask.**

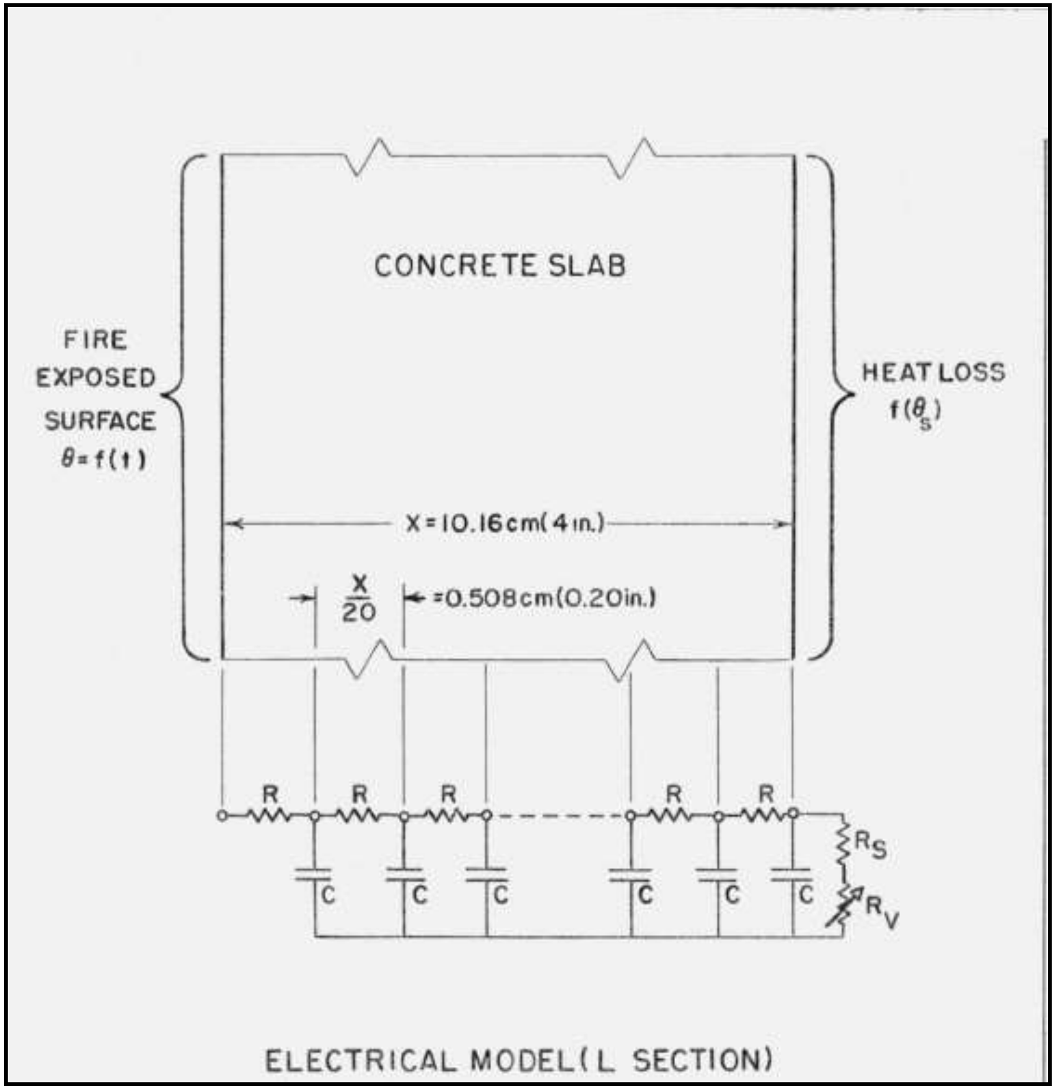
- **In practice, a commercial oscilloscope was used in conjunction with a photomultiplier tube as the generator.**
- **A special impedance converter was built to modify the balanced high-impedance signal applied to the deflection plates into a low-impedance single-ended output signal suitable for application to the model.**

- The electrical model was custom-made for each thermal situation being simulated.
- The following figure shows a photograph of one of the models used .



View of an electrical model

- It consists of a series of resistors representing the thermal resistance of the prototype and a group of capacitors shunting these resistors to ground and representing the heat capacity of the thermal system.
- The assembled model assumes the form of a plug-in unit, provisions being made for 20 discharge points.



Method of development of electrical model for a fire-exposed concrete slab.

THREE-DIMENSIONAL PHOTOELASTICITY

- Three-dimensional photoelasticity by the stress-freezing method has been considered to be one of the most powerful methods of experimental stress analysis.
- Its principles were well established more than thirty years ago, but its use outside the academic world has steadily declined.
- The reasons are cost and time needed to generate the desired information.

- This summarizes fifteen year's effort to develop stress freezing photoelasticity into a responsive and inexpensive tool for stress analysis in the industrial environment.
- The whole procedure of stress-freezing photoelasticity is reviewed and evaluated from a cost-effectiveness point of view.

- Techniques of three-dimensional photoelasticity are based upon the property of certain photoelastic plastics.
- A model made from such material is heated to its stress-annealing temperature;
  - it is stressed
  - usually by deadweight loading; and
  - it is cooled slowly with the weights still applied.

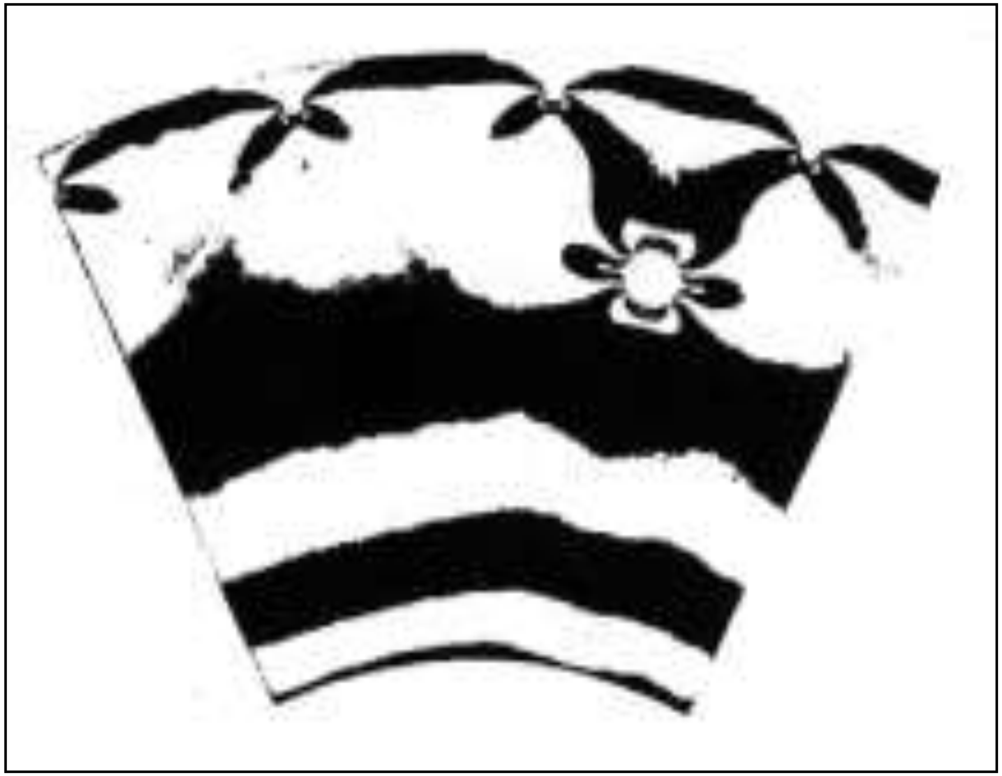
- The loaded model deforms at the elevated temperature, and this *elastic* deformation remains fixed during cooling.
- At room temperature, the weights are removed, but a mayor portion of the deformation remains locked in the model.
- Significantly, a photoelastic pattern corresponding to the elastic state of stress remains fixed in the model together with the deformation.

- This is called the *frozen-stress phenomenon, for the photoelastic pattern is "frozen" into* the model during the slow-cooling process.
- It is a remarkable fact that such models can be cut into any number of elements without disturbing the photoelastic patterns.

- **Accordingly, slabs are normally sawed out of three-dimensional models and analyzed individually in the manner of two-dimensional photoelasticity.**
- **The mechanical and optical anisotropy remains permanently fixed in the model, and with suitable materials, isochromatic patterns remain unchanged after several years of storage.**

- The following Figures illustrate an application of the frozen-stress technique.
- A photoelastic model used for a three-dimensional centrifugal stress study.
- The model was machined from a solid, stress-free slab of epoxy plastic; the entire circular body was produced, but the portion shown here is the part remaining after sections were cut out for analysis.

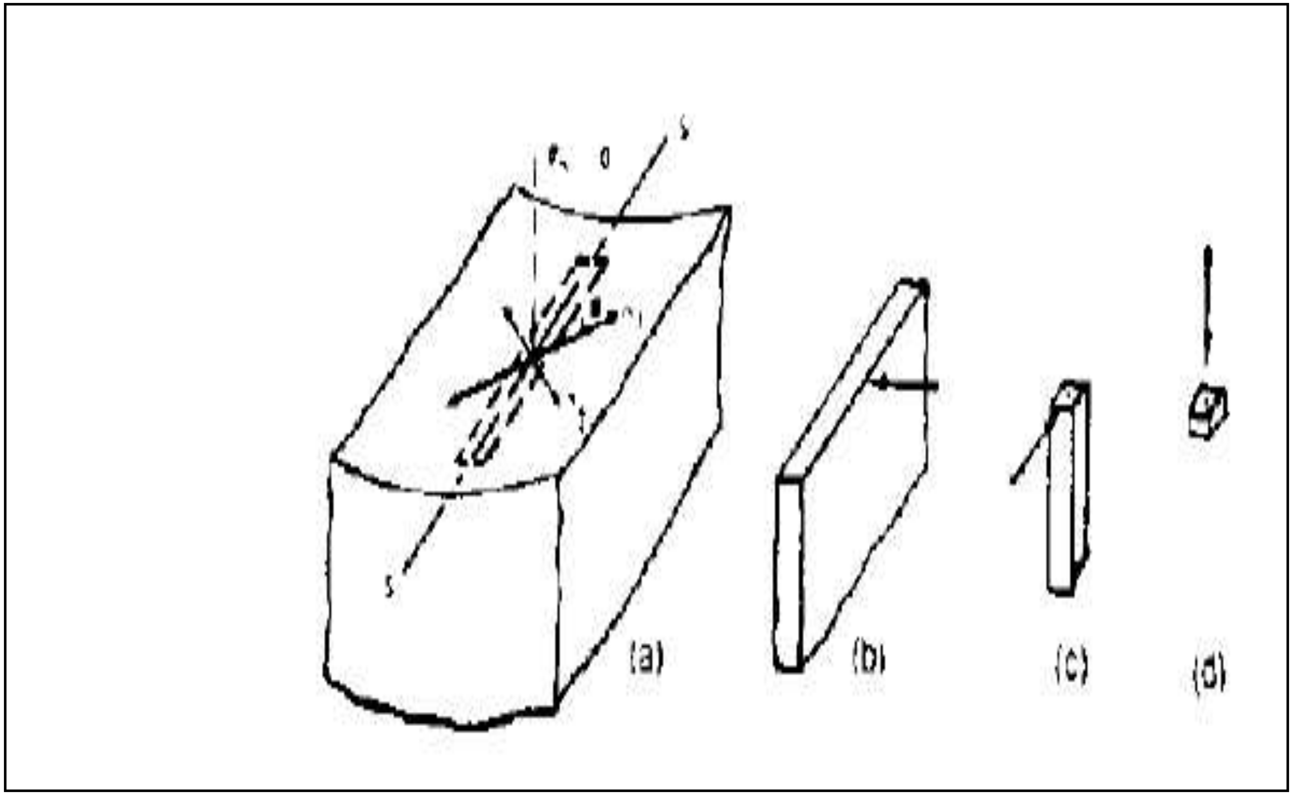
Photoelastic model for three-dimensional centrifugal-stress study



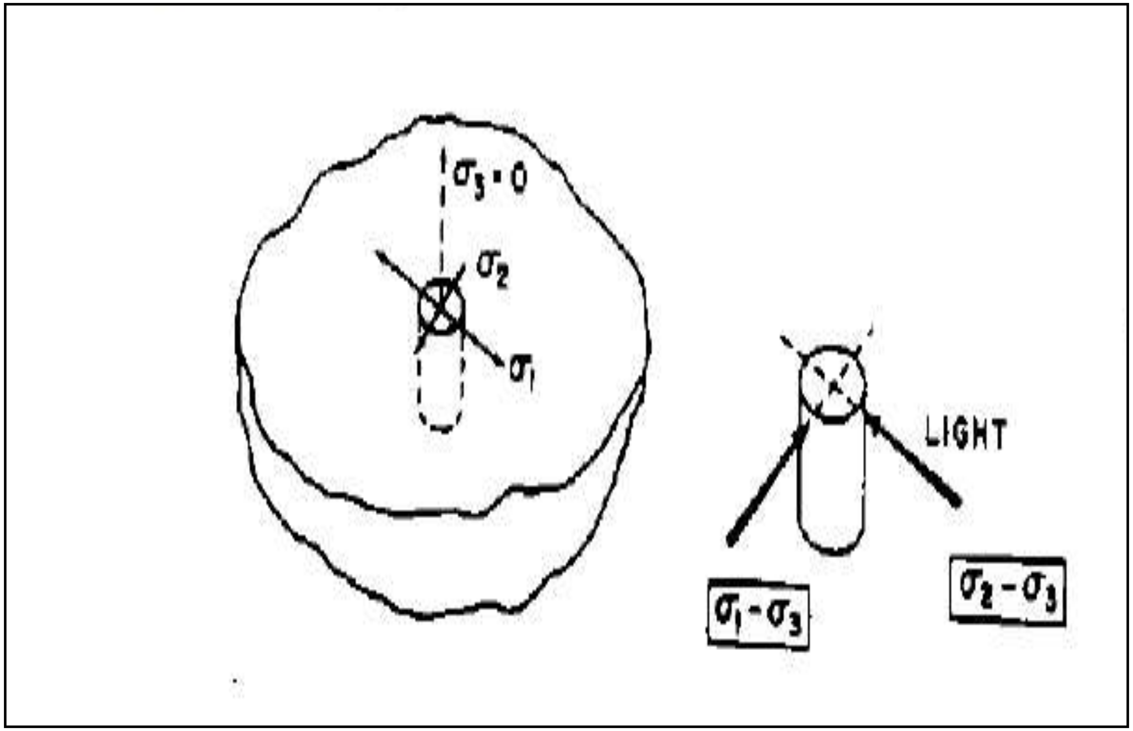
Isochromatic pattern for central slice

- **The above Figure shows the isochromatic pattern for a slice taken from this model along its central plane.**
- **Stress amplification resulting from the discontinuities in the geometry of the part become clearly evident.**
- **Since this is a section taken along a plane of symmetry, two principal stresses at every point lie in this plane.**

- Along the boundaries of the hole and slots, one of these principal stresses is zero, and the other — the tangential stress — is directly proportional to the isochromatic fringe order.
- Elsewhere, the isochromatic pattern yields the difference of principal stresses lying in the plane of the slice.
- In most three-dimensional analyses, the stresses along the surface of the part are of primary interest, since structural failure usually originates at the surface.



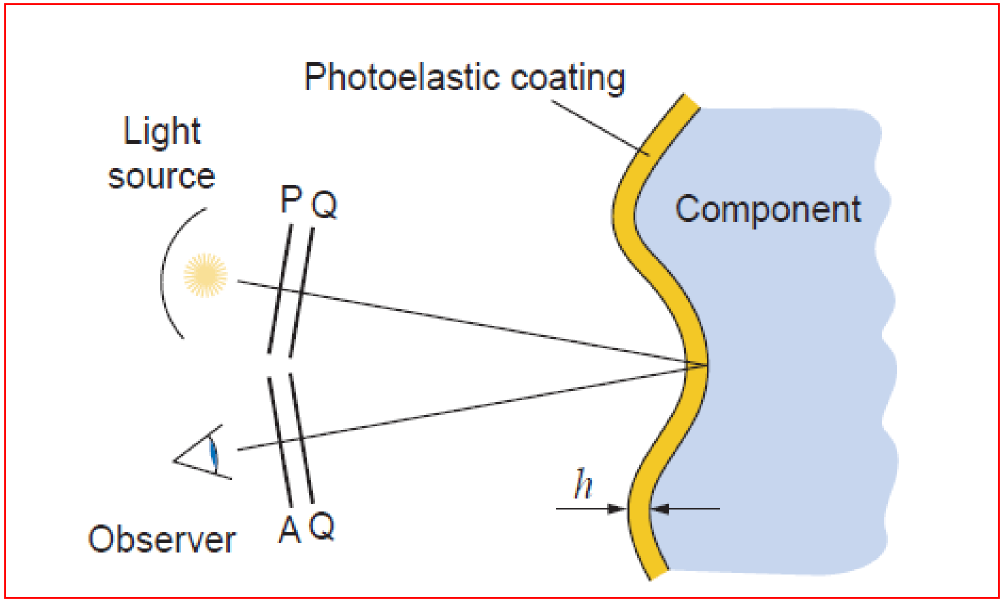
Steps in the analysis of surface stresses



Stress system at the surface of a core specimen

Photoelastic coatings

- Photoelasticity with an important application that permits surface analysis of actual components that have irregular surfaces.
- A photoelastic coating can be molded to the surface contour of a complicated part and bonded to it (Fig).



Photoelastic coating

- **Light is reflected at the coating–component interface and therefore propagates twice through the coating thickness *h, giving an effective* path length of 2*h in the coating.***
- **The component is now the primary load carrying member, not the photoelastic material.**

- **The in-plane coating strains are assumed to be equal to the in-plane surface strains in the component, and the analysis of the photoelastic patterns is based on the principal *strain difference, which is related to* the principal *stress difference in the component* through the elastic constants of the component material.**

- **In this respect, photoelastic coatings are regarded in much the same way as brittle coatings.**
- **As discussed in Dally and Riley (1991), compromises must be made.**
- **For example, the coating must be thick enough to generate a reasonable number of fringes in response to the component strains,**

- **yet not so thick that the average strains in the coating deviate significantly from the interface strains and the coating begins to reinforce the component.**
- **For components that undergo very small strains when loaded, very sensitive coatings must be used, and a careful analysis of the colors of the isochromatics may be needed to determine fractional fringe orders.**

Photostress Techniques

- PhotoStress is a widely used full-field technique for accurately measuring surface strains to determine the stresses in a part or structure during static or dynamic testing.
- With the PhotoStress method, a special strain-sensitive plastic coating is first bonded to the test part.

- Then, as test or service loads are applied to the part, the coating is illuminated by polarized light from a reflection polariscope.
- When viewed through the polariscope, the coating displays the strains in a colorful, informative pattern which immediately reveals the overall strain distribution and pinpoints highly strain areas.

- With an optical transducer (compensator) attached to the polariscope, quantitative stress analysis can be quickly and easily performed.
- Permanent records of the overall strain distribution can be made by photography or by video recording.

With PhotoStress:

- Instantly identify critical areas, highlighting overstressed and under stressed regions.
- Accurately measure peak stresses and determine stress concentrations around holes, notches, fillets, and other potential failure sites.

- **Optimize the stress distribution in parts and structures for minimum weight and maximum reliability.**
- **Measure principal stresses and directions at any point on the coated part.**
- **Test repeatedly under varying load conditions, without recoating the part.**

- Make stress measurements in the laboratory or in the field — unaffected by humidity or time.
- Identify and measure assembly stresses and residual stresses.
- Detect yielding, and observe redistribution of strains in the plastic range of deformation.

- PhotoStress coatings can be applied to the surface of any test part of its shape, size, or material composition.
- For coating complex shapes, liquid plastic is cast on a flat-plate mold and allowed to partially polymerize.
- When fully cured, the plastic coating is bonded in place with special reflective cement, and the part is then ready for testing.

- PhotoStress has an established history of successful applications in every field of manufacture and construction where stress analysis is employed, including:
- automotive, farm machinery, aircraft and aerospace, building construction, engines, pressure vessels, shipbuilding, office equipment, bridges, appliances, plus many others.

- PhotoStress offers the following types of analysis and measurements:
- 1. Full-field analysis, permitting overall assessment of strain/stress magnitudes and gradients, and areas of maximum stress.
- 2. Quantitative measurements expressed in stress units (MPa) or in strain units (mm/mm):

- a. The directions of principal stresses at all points on the surface of the structure.
- b. The magnitude and sign of the tangential stress along free boundaries, and in all regions where the state of stress is uniaxial.

- c. In a biaxial stress state, the magnitude of the difference in principal stresses at any selected point on the coated surface of the test object.
- d. Individual values and sign of principal stresses by the PhotoStress slitting method.

Special Features:

- Immediate recognition of stress gradients and overall stress distribution.
- Immediate identification of overstressed and under stressed areas.
- Observation of stress distribution under varying modes of loading.

- **Comparison between the actual stress distribution obtained by PhotoStress analysis and the modeling analysis by Finite Element (FEA).**
- **Areas of yielding (elastoplastic deformations) can be identified and measured after the part is unloaded, by observing and measuring the residual color pattern.**

PHOTOELASTICITY

- Photoelasticity is an experimental technique for stress and strain analysis that is particularly useful for members having complicated geometry, complicated loading conditions, or both.
- The name photoelasticity reflects the nature of this experimental method:

- ***photo implies*** **the use of light rays and optical techniques, while *elasticity depicts the study of stresses and* deformations in elastic bodies.**
- **Photoelastic analysis is widely used for problems in which stress or strain information is required for extended regions of the structure.**

- **It provides quantitative evidence of highly stressed areas and peak stresses at surface and interior points of the structure.**

PHOTOELASTIC BEHAVIOR

- **The photoelastic method is based upon a unique property of some transparent materials, in particular, certain plastics / polymers.**

History

- The photoelastic phenomenon was first described by the Scottish physicist David Brewster.
- Photoelasticity developed at the beginning of the twentieth century with the works of E.G.Coker and L.N.G. Filon of University of London.

- **Their book *on Photoelasticity* published in 1930 by the Cambridge Press became a standard text on the subject.**
- **Between 1930 and 1940 many other books: Russian, German and French appeared on the subject.**
- **At the same time much development occurred in the field.**

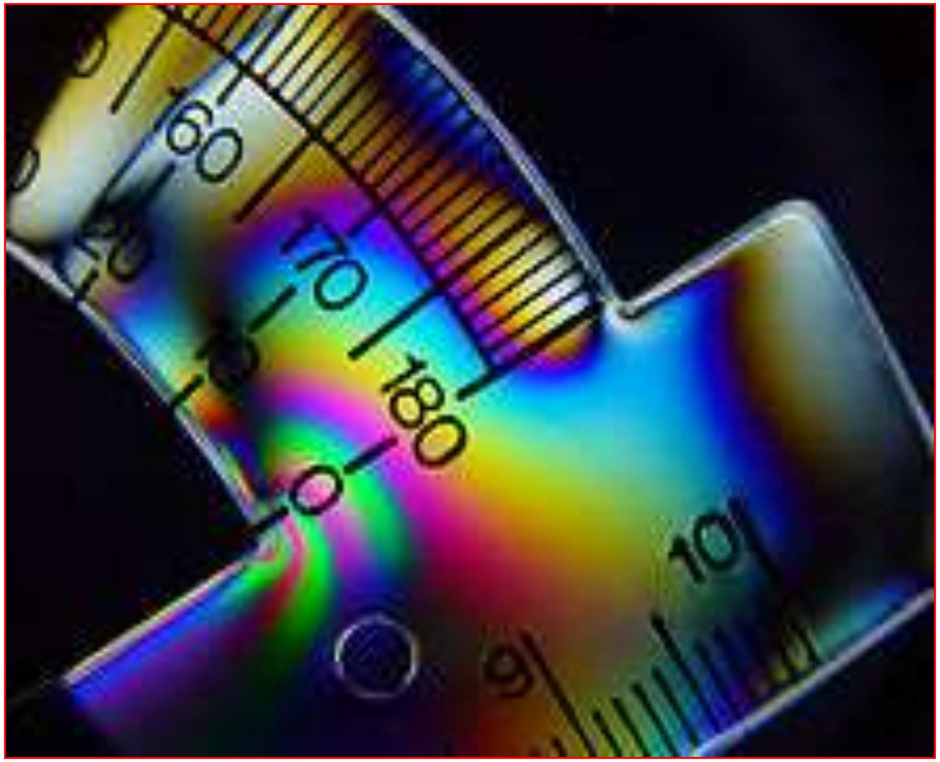
- **Great improvements were achieved in the technique and the equipment was simplified.**
- **With the improvement in technology the scope of photoelasticity was extended to three-dimensional state of stress.**
- **Many practical problems were solved using photoelasticity, and it soon became popular.**

- **A number of photoelastic laboratories were established in educational institutions and industries.**
- **With the advent of digital polariscope using light-emitting diodes, continuous monitoring of structures under load became possible.**
- **This led to the development of dynamic photoelasticity.**

- Dynamic photoelasticity has contributed greatly to the study of complex phenomena such as fracture of materials.

- Consider a model of some structural part made from a photoelastic material.
- When the model is stressed and a ray of light enters along one of the directions of principal stress, a remarkable thing happens.
- The light is divided into two component waves:

- each with its plane of vibration (plane of polarization) parallel to one of the remaining two principal planes (planes on which shear stress is zero).
- Furthermore, the light travels along these two paths with different velocities, which depend upon the magnitudes of the remaining two principal stresses in the material.



- **Tension lines in plastic protractor seen under cross-polarized light.**

- **The method is based on the property of birefringence, as exhibited by certain transparent materials.**
- **Birefringence is the phenomenon in which a ray of light passing through a birefringent material experiences two refractive indices.**
- **The property of birefringence (or double refraction) is observed in many optical crystals.**

- **Photoelastic materials exhibit the property of birefringence, and the magnitude of the refractive indices at each point in the material is directly related to the state of stresses at that point.**
- **Information such as maximum shear stress and its orientation are available by analyzing the birefringence with an instrument called a polariscope.**

- **The following methods may be used to produce plane-polarized light:**
- **Blackened glass plate**
- **Pile of plates**
- **Calcite Crystal**
- **The Nicol Prism**
- **The Glan Thompson Polarizer**
- **Ahrens Polarizer**

- **When a ray of light passes through a photoelastic material, its electromagnetic wave components are resolved along the two principal stress directions and each component experiences a different refractive index due to the birefringence.**
- **The difference in the refractive indices leads to a relative phase retardation between the two components.**

- **Assuming a thin specimen made of isotropic materials, where two-dimensional photoelasticity is applicable, the magnitude of the relative retardation is given by the *stress-optic law*:**

$$\boxed{\Delta = \frac{2\pi t}{\lambda} C(\sigma_1 - \sigma_2)}$$

- **Where,**
- **Δ is the induced retardation,**
- ***C* is the stress-optic coefficient,**
- ***t* is the specimen thickness,**
- ***λ* is the vacuum wavelength, and**
- **σ_1 and σ_2 are the first and second principal stresses, respectively.**
- **The retardation changes the polarization of transmitted light.**

Stress-Optic Law

- Maxwell reported in 1853 that the changes in the indices of refraction were linearly proportional to the loads (thus to the stresses or strains for a linearly elastic material) and followed the relationship:

$$\boxed{\left.\begin{aligned} n_1 - n_0 &= C_1\sigma_1 + C_2(\sigma_2 + \sigma_3) \\ n_2 - n_0 &= C_1\sigma_2 + C_2(\sigma_3 + \sigma_1) \\ n_3 - n_0 &= C_1\sigma_3 + C_2(\sigma_1 + \sigma_2) \end{aligned}\right\}}$$

- **where σ_1, σ_2, σ_3 = principal stresses at the point**
- **n_0 = index of refraction of material in the unstressed state.**
- **n_1, n_2, n_3= principal refractive indices of the material in the stressed state associated with the principal stresses, σ_1, σ_2 and σ_3 respectively.**
- **C_1, C_2 = stress-optic coefficients, which depend on the material.**

- **Equations are the fundamental relationships between stress and optical effect and are known as the stress-optic law.**
- **Eliminating n_0 from equations, we get**

$$
\boxed{\left.\begin{aligned}
n_2 - n_1 &= (C_2 - C_1)(\sigma_1 - \sigma_2) = C(\sigma_1 - \sigma_2) \\
n_3 - n_2 &= (C_2 - C_1)(\sigma_2 - \sigma_3) = C(\sigma_2 - \sigma_3) \\
n_1 - n_3 &= (C_2 - C_1)(\sigma_3 - \sigma_1) = C(\sigma_3 - \sigma_1)
\end{aligned}\right\}}
$$

- where $C = C_2 - C_1$ is the relative or differential stress-optic coefficient expressed in terms of Brewster's (1 Brewster = 10^{-12} cm^2/dyn = 10^{-12} m^2/N).
- Now the wave equation is,

$$\boxed{\begin{aligned} E &= \alpha \cos \frac{2\pi}{\lambda}(z - ct) \\ &= \alpha \cos \phi \end{aligned}}$$

- **Angular phase shift between two waves, $\Delta = \phi_2 - \phi_1$**
- **Since the stressed photoelastic models behaves like temporary wave plate, hence,**

$$\phi_1 = \frac{2\pi h}{\lambda}(n_1 - n_0)$$

$$\phi_2 = \frac{2\pi h}{\lambda}(n_2 - n_0)$$

$$\therefore \quad \Delta = \phi_2 - \phi_1 = \frac{2\pi h}{\lambda}(n_2 - n_1)$$

- **Therefore, if a beam of plane-polarized light is passed through a slice of thickness h at normal incidence, the relative retardation Δ accumulated along each of the principal stress directions becomes**

$$\boxed{\begin{aligned} \Delta_{12} &= \frac{2\pi hC}{\lambda}(\sigma_1 - \sigma_2) \\ \Delta_{23} &= \frac{2\pi hC}{\lambda}(\sigma_2 - \sigma_3) \\ \Delta_{31} &= \frac{2\pi hC}{\lambda}(\sigma_3 - \sigma_1) \end{aligned}}$$

- **where Δ_{12}, Δ_{23}, Δ_{31} is the magnitude of the relative retardation developed between components of light beam propagating in the σ_3, σ_1, σ_2 directions respectively.**
- **For two-dimensional or plane-stress problems ($\sigma_3 = 0$) and we get**
- $$\Delta = \frac{2\pi hC}{\lambda}(\sigma_1 - \sigma_2)$$

- The polariscope combines the different polarization states of light waves before and after passing the specimen.
- Due to optical interference of the two waves, a fringe pattern is revealed.
- The number of fringe order *N* is denoted as

$$\boxed{N = \frac{\Delta}{2\pi}}$$

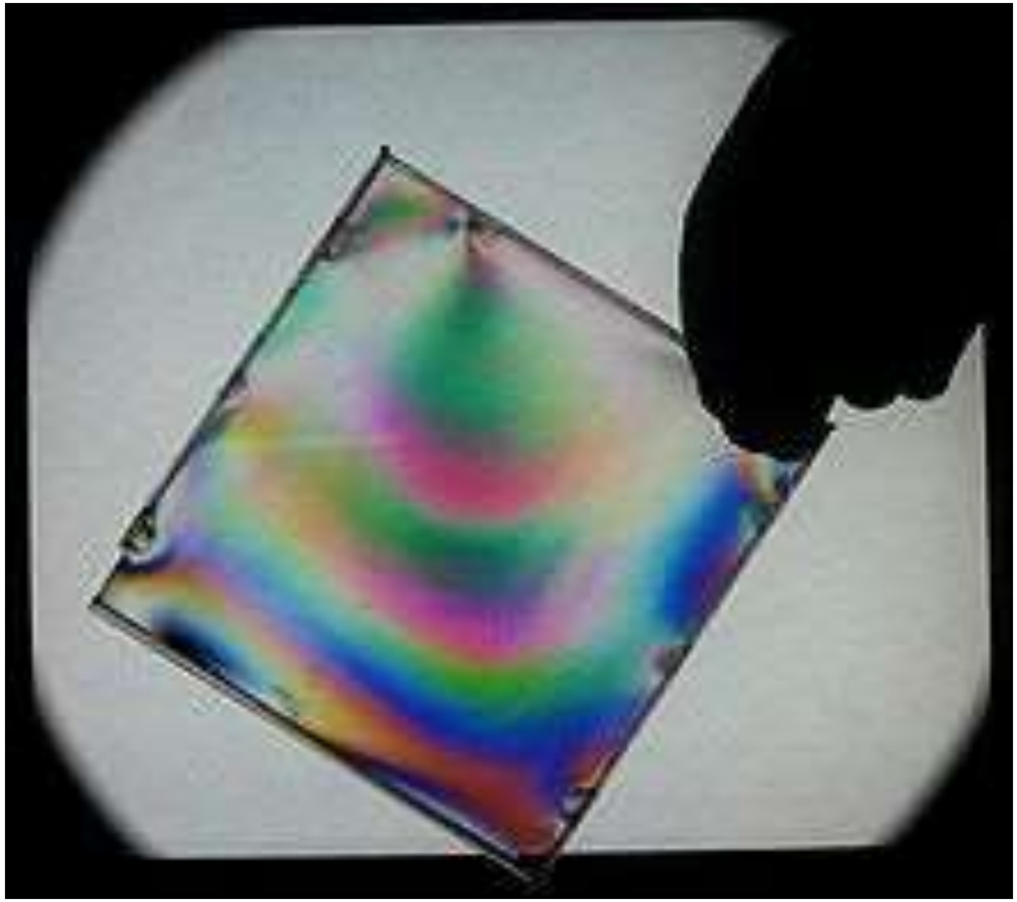
- **which depends on relative retardation.**
- **By studying the fringe pattern one can determine the state of stress at various points in the material.**
- **For materials that do not show photoelastic behavior, it is still possible to study the stress distribution.**

- The first step is to build a model, using photoelastic materials, which has geometry similar to the real structure under investigation.
- The loading is then applied in the same way to ensure that the stress distribution in the model is similar to the stress in the real structure.

Isoclinic's and Isochromatics

- **Isoclinics** are the loci of the points in the specimen along which the principal stresses are in the same direction.
- **Isochromatics** are the loci of the points along which the difference in the first and second principal stress remains the same.
- **Thus they are the lines which join the points with equal maximum shear stress magnitude.**

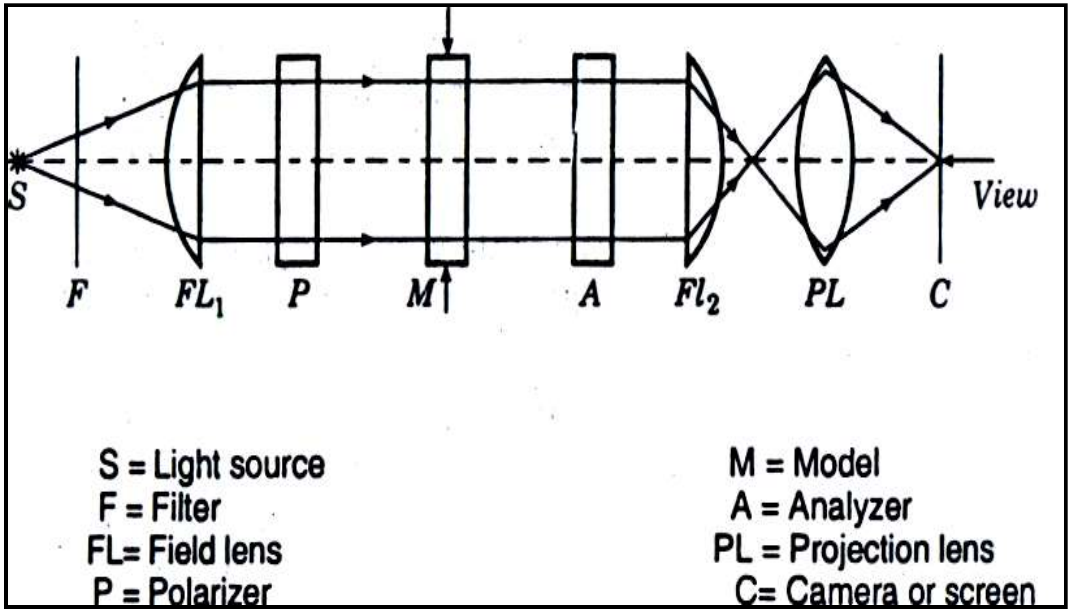
Two-dimensional photoelasticity



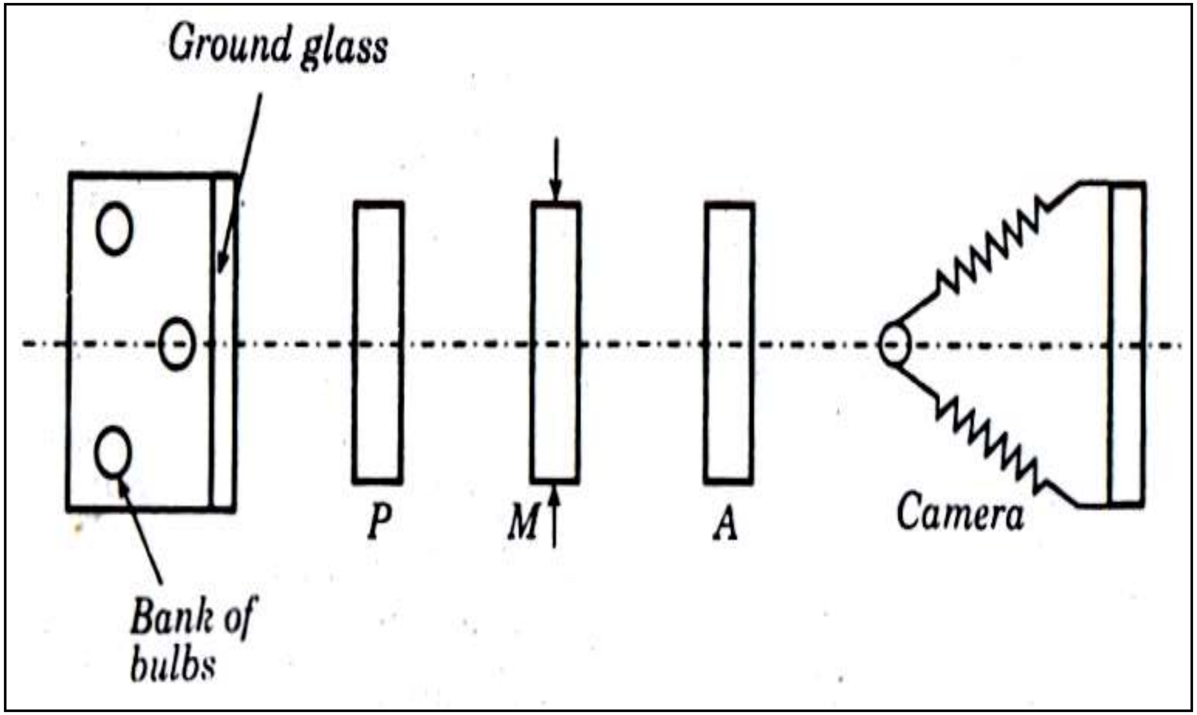
- Photoelasticity can be applied both to three - dimensional and two - dimensional state of stress.
- But the application of photoelasticty to the three-dimensional state of stress is more involved as compared to the state of two-dimensional or plane-stress system.
- So the present section deals with application of photoelasticity in investigation of a plane stress system.

- **The two basic kinds of setup used are plane polariscope and circular polariscope.**
- **In the plane polariscope, plane-polarized light is used and in the circular polariscope, circularly polarized light is used.**

- **When the light is transmitted through the model then the polariscope is called of the transmission type.**
- **The polariscope may also be either of the lens type or diffused light type.**
- **The basic arrangement of a lens type plane polariscope is shown in figure shows the set up for a diffused light polariscope.**



Lens type plane polariscope



Diffused light plane polariscope

- **The working principle of two-dimensional photoelasticity allows the measurement of retardation, which can be converted to the difference between the first and second principal stress and their orientation.**

- **To further get values of each stress component, a technique called stress-separation is required.**
- **Several theoretical and experimental methods are utilized to provide additional information to solve individual stress components.**

Plane polariscope

- The setup consists of two linear polarizers and a light source.
- The light source can either emit monochromatic light or white light depending upon the experiment.
- First the light is passed through the first polarizer which converts the light into plane polarized light.

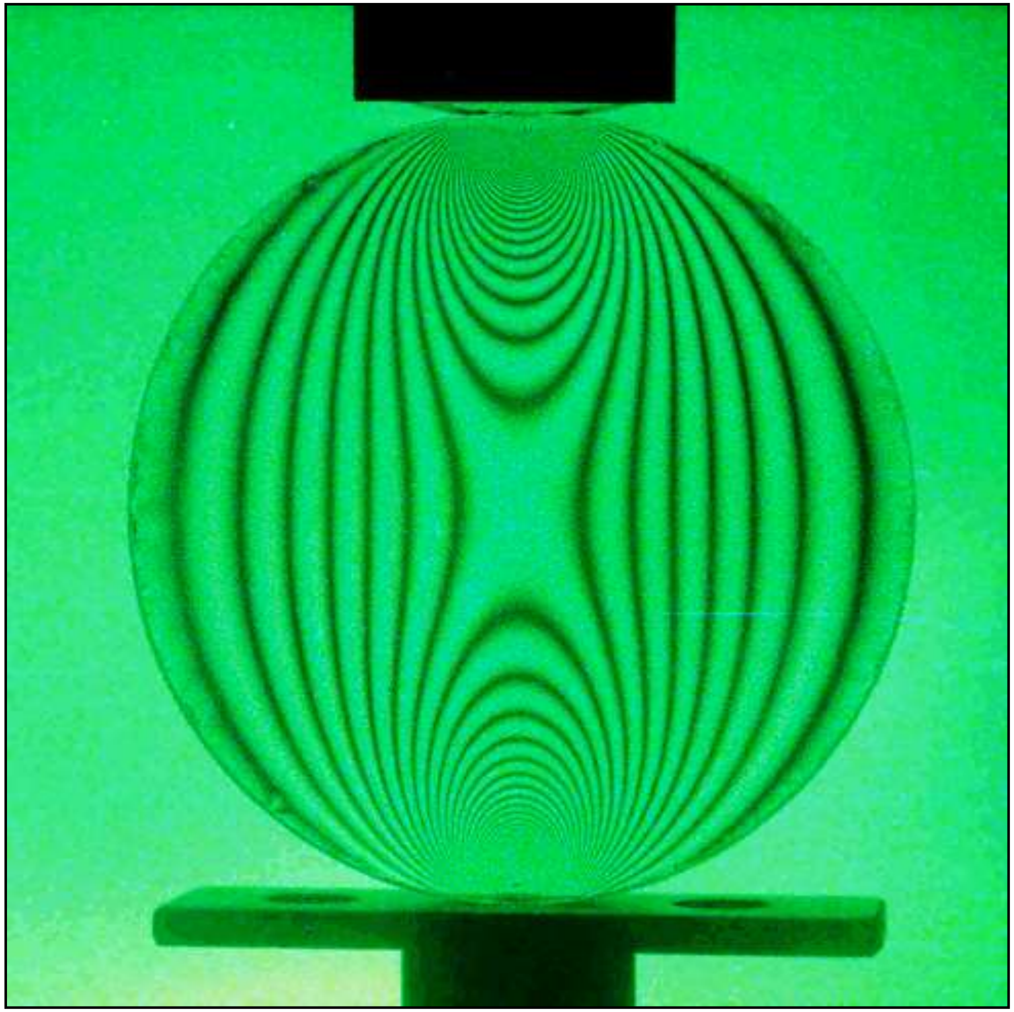
- The apparatus is set up in such a way that this plane polarized light then passes through the stressed specimen.
- This light then follows, at each point of the specimen, the direction of principal stress at that point.
- The light is then made to pass through the analyzer and we finally get the fringe pattern.

- The fringe pattern in a plane polariscope setup consists of both the isochromatics and the isoclinics.
- The isoclinics change with the orientation of the polariscope while there is no change in the isochromatics.
- The same device functions as a plane polariscope when quarter wave plates are taken aside or rotated so their axes parallel to polarization axes.

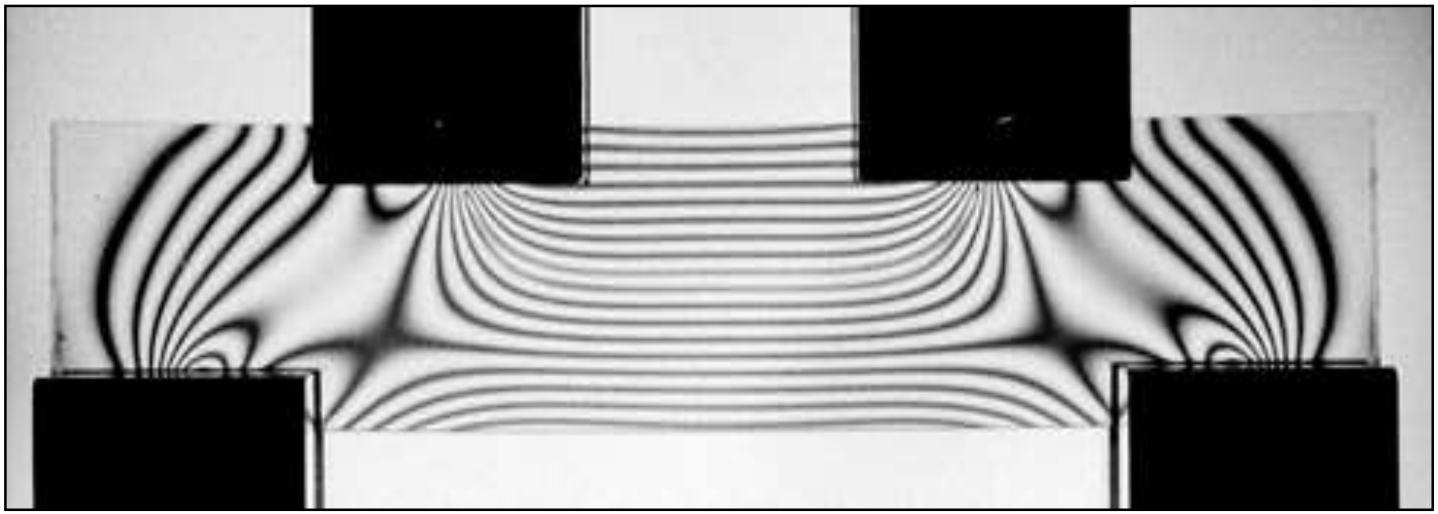
- **In a circular polariscope setup two quarter-wave plates are added to the experimental setup of the plane polariscope.**
- **The first quarter-wave plate is placed in between the polarizer and the specimen and the second quarter-wave plate is placed between the specimen and the analyzer.**

- **The effect of adding the quarter-wave plate after the source-side polarizer is that we get circularly polarized light passing through the sample.**
- **The analyzer-side quarter-wave plate converts the circular polarization state back to linear before the light passes through the analyzer.**

- The basic advantage of a circular polariscope over a plane polariscope is that in a circular polariscope setup we only get the isochromatics and not the isoclinics.
- This eliminates the problem of differentiating between the isoclinics and the isochromatics.



- **Light-field isochromatics in a diametrally loaded circular disk**



Beam in 4-point bending

- **Effect of a Stressed Model in a Plane Polariscope**
- **Dark-Field set up**
- **Consider the dark-field set up of the plane polariscope, when the polarizer and analyzer are crossed.**
- **Bright-Field set up**
- **In the bright-field set up the axis of the analyzer is parallel to that of the polarizer.**

- In order to achieve higher accuracy, as is desirable in many applications, the following methods may be used:
- Compensation techniques
- Colour matching techniques
- Equidensometry method

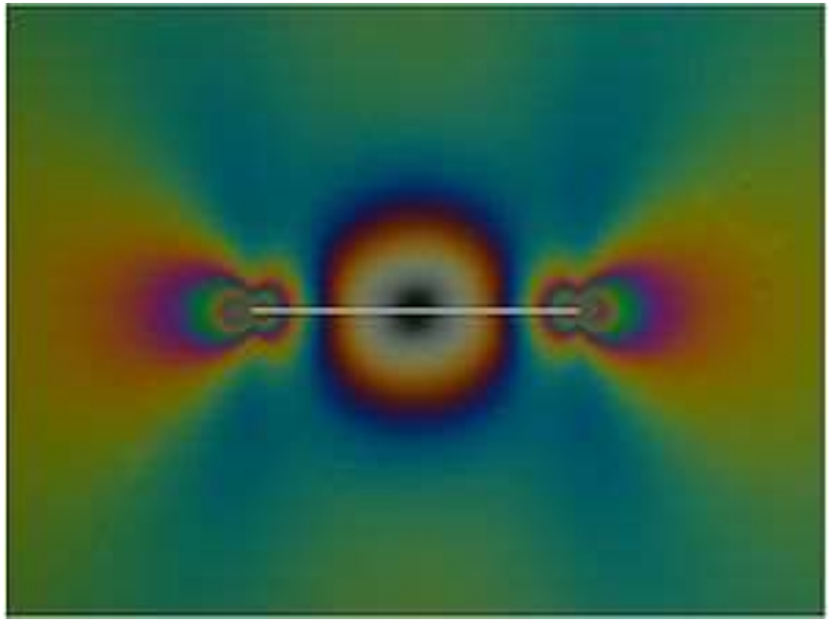
- Compensation is a technique in which partial modification of relative retardation either by addition or subtraction is brought about so that the fractional fringe order at a point become integral.
- Then by knowing the amount of relative retardation added or subtracted the actual fringe order at that point can be ascertained.
- The following methods for compensation techniques are most commonly used:

- The Babinet compensation method
- The Babinet Soleil compensation method
- Tension or compression strip method
- Tardy method of compensation
- Senarmont method of compensation
- Photometric method

Applications

- Photoelasticity has been used for a variety of stress analyses and even for routine use in design, particularly before the advent of numerical methods, such as for instance finite elements or boundary elements.

- **Digitization of polariscopy enables fast image acquisition and data processing, which allows its industrial applications to control quality of manufacturing process for materials such as glass and polymer.**
- **Dentistry utilizes photoelasticity to analyze strain in denture materials.**



- **Photoelastic model to validate the [stiffener] model. Isochromatic fringe patterns around a steel platelet in a photo-elastic two-part epoxy resin.**

- Photoelasticity can be used to investigate the highly localized stress state within masonry or in proximity of a rigid line inclusion (stiffener) embedded in an elastic medium.
- Dynamic photoelasticity integrated with high-speed photography is utilized to investigate fracture behavior in materials.

- **Another important application of the photoelasticity experiments is to study the stress field around bi-material notches.**
- **Bi-material notches exist in many engineering application like welded or adhesively bonded structures.**

- ***Advantages—Photoelasticity, as used for two dimensional*** **plane problems,**
- **provides reliable full-field values of the difference between the principal normal stresses in the plane of the model**
- **provides uniquely the value of the non vanishing principal normal stress along the perimeter(s) of the model, where stresses are generally the largest**

- furnishes full-field values of the principal stress directions (sometimes called stress *trajectories)*
- is adaptable to both static and dynamic investigations
- requires only a modest investment in equipment and materials for ordinary work
- is fairly simple to use

- **Disadvantages:**—*On the other hand, photoelasticity*
- requires that a model of the actual part be made (unless photoelastic coatings are used)
- requires rather tedious calculations in order to separate the values of principal stresses at a general interior point

- **can require expensive equipment for precise analysis of large components**
- **is very tedious and time-consuming for three dimensional work**

Electrical Resistance Strain Gauges

- In the electrical resistance strain gauges the displacement or strain is measured as a function of the resistance change produced by the displacement in the gauging circuit.
- An ideal strain gauge should have the following basic characteristics:

- **The gauge should be of extremely small size (gauge length and width) so as to adequately estimate strain at a point.**
- **The gauge should be of significant mass to permit the recording of dynamic strains.**
- **The gauge should be easy to attach to the member being analysed and easy to handle.**

- The strain sensitivity and accuracy of the gauge should be sufficiently high.
- The gauge should be unaffected by temperature, vibration, humidity or other ambient conditions.
- The calibration constant for the gauge should be stable over a wide range of temperature and time.

- The gauge should be capable of indicating both static and dynamic strains.
- It should be possible to read the gauge either on location or remotely.
- The gauge should exhibit linear response to strain.
- The gauge and the associated equipment should be available at a reasonable cost.

- **The gauge should be suitable for use as a sensing element or other transducer systems.**

Types of Resistance Strain Gauges

There are basically four types of electrical resistance strain gauges as classified below:

- **Unbonded gauges: Non-metallic & Metallic**
- **Bonded gauges: Non-metallic & Metallic**
  - **(i) Wire type**
  - **(ii) Foil type**
- **Weldable gauges**
- **Piezoresistive gauges**

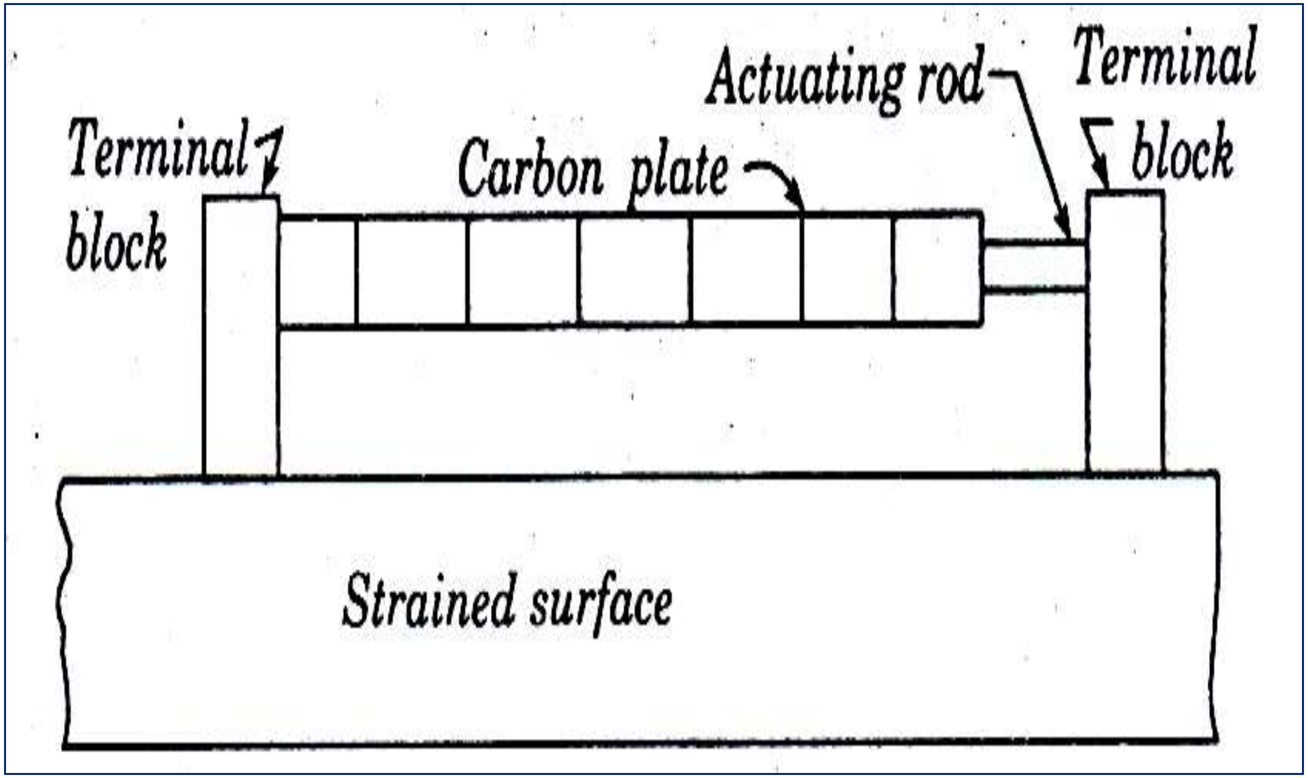
- In 1856 Lord Kelvin reported that the electrical resistance of certain wires varied with the tension to which the wires were subjected.
- Bridgeman in 1923 confirmed Kelvin's results in a series of tests involving wires under hydrostatic pressure.
- After 1930, when attempts were made to apply the phenomenon of strain sensitivity in wires to the actual measurement of strain in other bodies.

- The first use of this principle for strain measurements was made by Carlson and Eaton about 1931.
- A non metallic, unbounded resistance gauge was however developed and used by Mc – Collum and Peters in 1924.
- A non – metallic, bonded resistance gauge was developed by Bloach in 1935.

- **The bonded wire metallic strain gauge was developed independently and almost simultaneously in 1938 by Simmons at the California Institute of Technology and Ruge at the Massachusetts Institute of Technology, now commercially known as the SR 4 gauges, and marketed by Baldwin – Lima – Hamilton Corporation of U.S.A.**

- During the 1950's considerable attention was given to the foil – type strain gauges.
- Currently the foil gauges have largely displacement the wire gauges.
- A uniquely constructed weldable wire filament strain gauge has been developed recently for application in many hostile environments and installation by Ailtech (U.S.A)

- **Unbonded - NonMetallic Gauges**
- **The unbonded nonmetallic gauge is a mechanically actuated gauge.**
- **When one part of the gauge is displaced with respect to another is developed a change in pressure on the measuring element of the gauge.**
- **This change in pressure changes the resistance of the element which may be recorded by electrical means.**



Unbonded non-metallic strain gauge

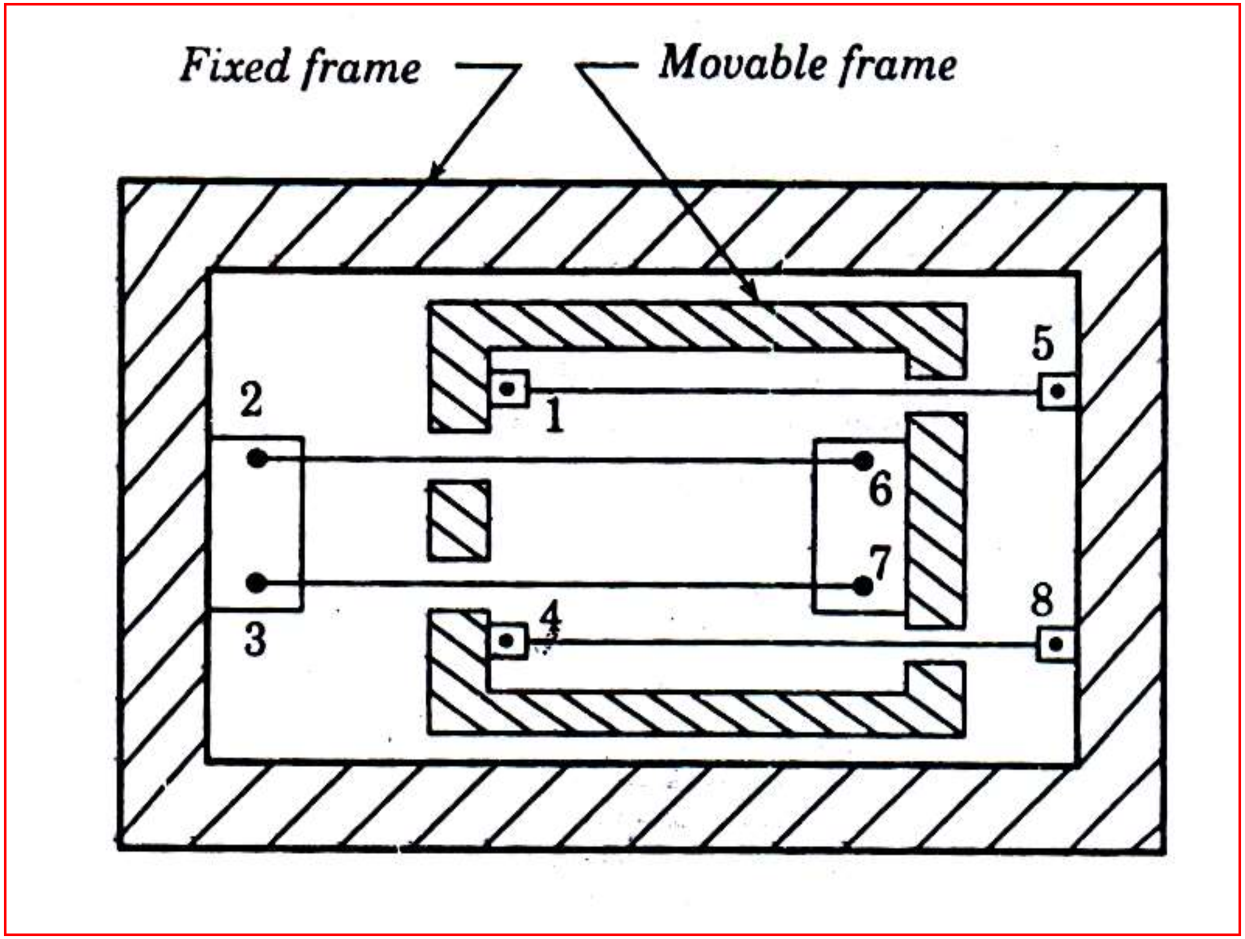
- **This gauge is composed of a series of carbon plates arranged in a stack.**
- **The stack is so adjusted that a displacement of one parts of the gauge relative to another changes the pressure, on the stack of plates.**
- **When the strain is applied in the structure to which the gauge is attached, the change in length is communicated to the carbon-plates.**

- **Gauge of this kind have been used to determine displacements, loads and strains in flexible cables, airplanes, bridges, vibrating members, dynamometers and pressure gauges.**
- **However, with the advancement of metallic gauges the usefulness of these types of gauges has reduced materially.**

- **Unbonded-Metallic Gauges**
- **The principle of the unbonded-metallic gauges is based on the change in electrical resistance of a metallic wire due to the change in tension of the wire.**
- **The first device of this kind was designed by Carlson and Eaton in 1930.**
- **This type of gauge is constructed by winding wire in three coils:**

- the first providing a coil unaffected by the gauge motion, and the other two having tensions altered by the gauge motion, each in an opposite manner.
- The whole is mounted in a sleeve that allows only longitudinal movement.
- The coils are placed under initial tension into a four arm Wheatstone bridge.

- **A gauge of this type is shown in Fig.**
- **These type of gauges are rarely used for experimental stress analysis.**
- **However, these type of gauges have been incorporated into accelerometers and pressure pickups.**

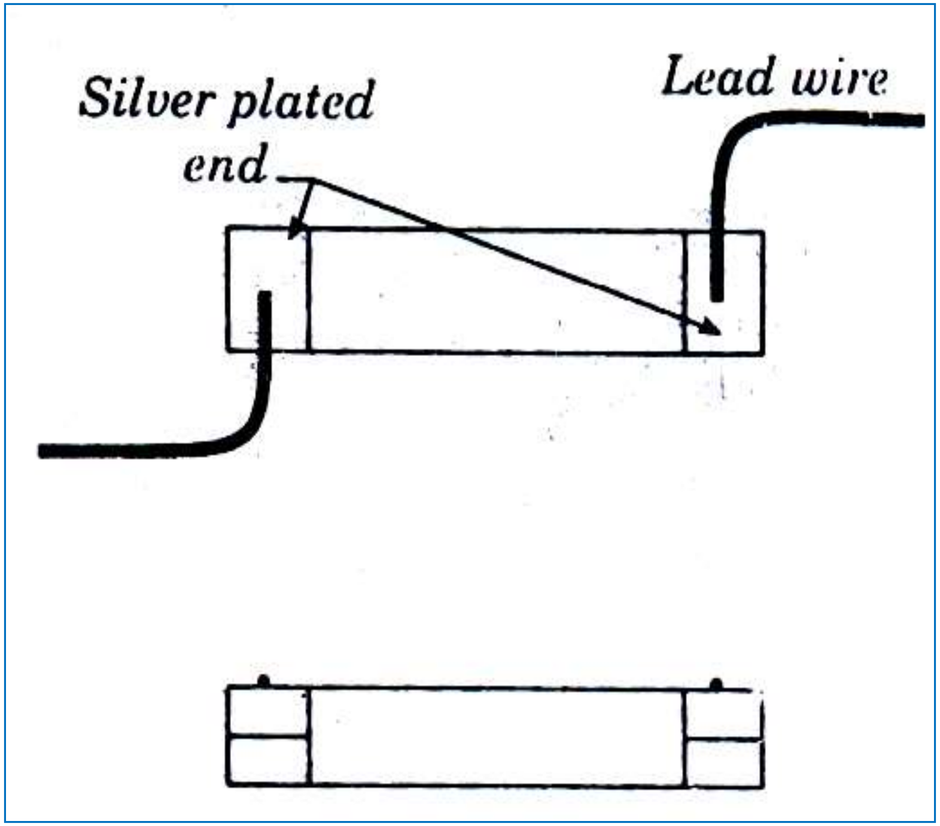


- **Unbonded metallic strain gauge**

- **Bonded Non-Metallic Gauges**
- **A strain gauge using direct bonding of a non-metallic resistor element to a material in which the strain is so to be measured was reported by Bloach in 1935.**
- **In this gauge a carbon coating is applied directly to the surface of the structure in which strain is to be measured.**

- For metallic structures the surface is first coated with a non-conducting material.
- Such a coating is stretched, the carbon particle would move apart, and the under-coating is compressed, the particles would move closer together, and the resistance will change.
- This resistance change can be interpreted in terms of strain.

- Generally these type of gauges are made by impregnating carbon particles in plastic sheets.
- These sheets are then cut into strips about 6 mm wide and 25 mm long.
- On each end of the strip a silver band is plated so that lead wires may be attached (fig).
- The gauge is bonded directly to the surface to be strained with a common glue.



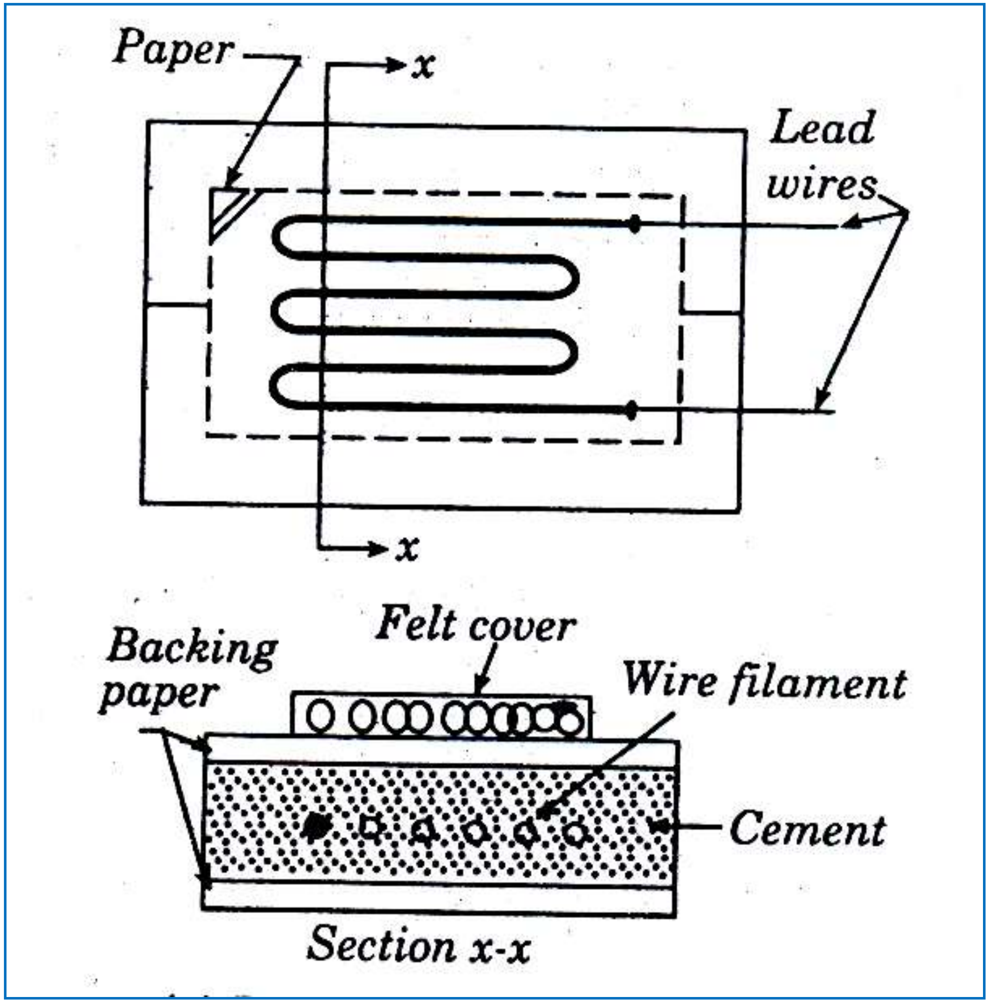
- **Bonded non-metallic strain gauge**

- These sensitivity and resistance of the gauge are affected by temperature and humidity.
- This gauge is of rugged construction and can withstand rough handling.
- However, the cross-sensitivity of the gauge is quite high.

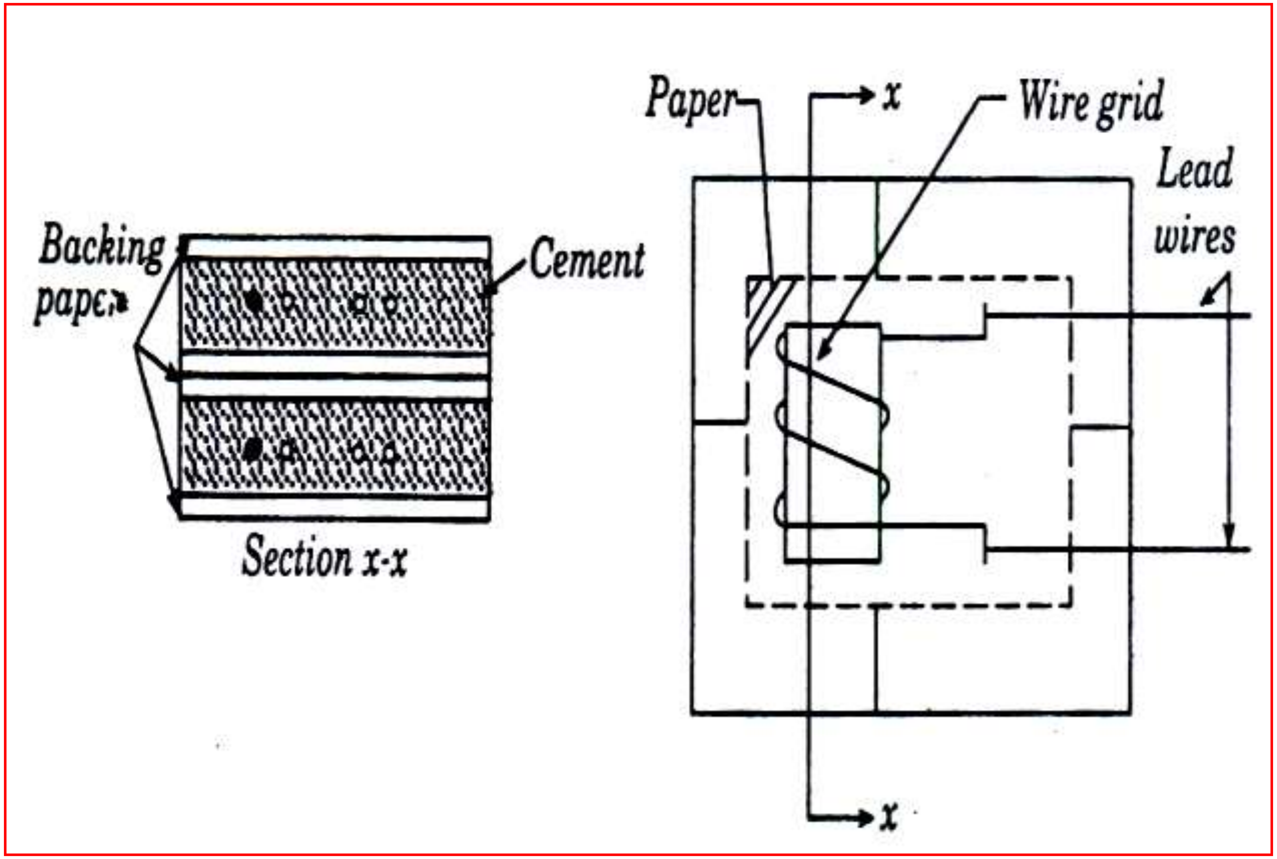
- **Bonded Metallic Gauges**
- The bonded metallic type of strain gauge consists of a length of a strain-sensitive conductor mounted on a small piece of paper or plastic backing.
- In use this gauge is cemented to the surface of the structural member to be tested.
- These gauges may be either of the wire or foil type.

- In the case of wire strain gauges, the filament consists of a long length of wire in the form of a grid fixed in place with a suitable cement.
- The wire grid may be either of the flat type (fig. a) or wrap-around type (Fig. b).
- After attaching the lead wires to the two ends of the grid, a second piece of paper is cemented over the wire as a cover.

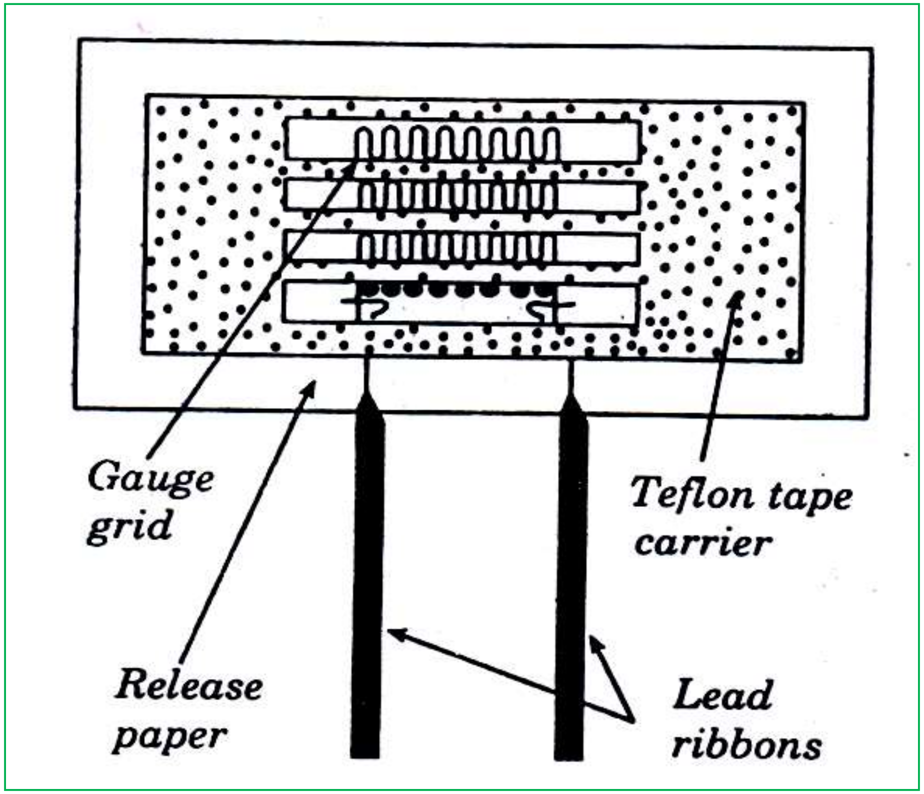
- In the wrap around type of wire gauges, the strain-sensitive wire is wound around a cylindrical core in the form of a close-wound helix.
- This core is then flattened and cemented between layers of paper for purpose of protection and insulation.
- Fig.(c) shows a flat wire grid free filament construction.



(a) Bonded wire flat grid gauge

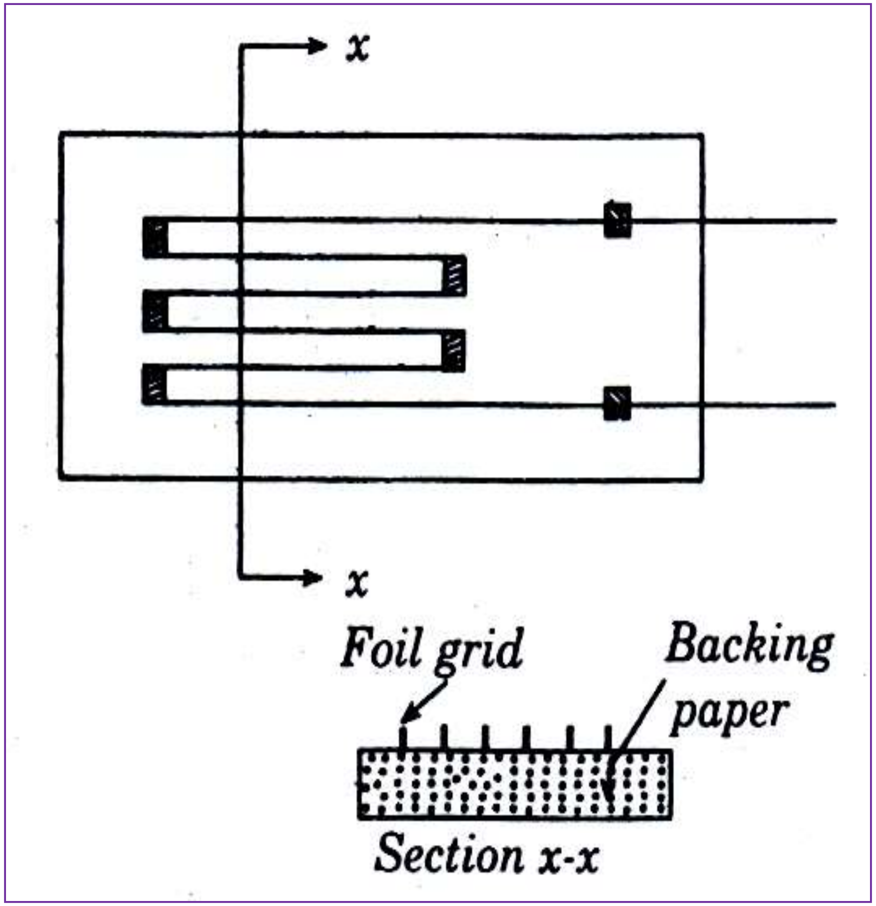


(b) Bonded wire wrap-around gauge



(c) Flat wire grid free filament construction

- The foil type of stain gauge has a grid made from a very thin strain-sensitive foil (fig d).
- The width of foil is very large as compared to the thickness so that the gauge provides a much larger area for cementing the gauge.



(d) Bonded flat foil grid gauge

- The gauge configuration is obtained by printing the desired pattern on a sheet of foil with acid resistant ink and subsequently etching away the unprotected metal.
- Another method of manufacture involves precision punching of the gauges from a foil sheet.
- The foil type of gauges has the following advantages over the wire type gauges.

- **The width of the foil at the end of each loop can be greatly increased to reduce the sensitivity of the gauge to transverse strains.**
- **The cross-section of the gauge conductor is rectangular, resulting in the high ratio of surface area to cross-section area.**
- **This increases heat dissipation and avoids adhesion between the grid and the backing material.**

- **The gauge factor is higher by 4 to 10 per cent that other gauges.**
- **These gauges are easier to manufacture.**
- **These gauges can be used to measure strain on curved surfaces.**
- **These gauges are suitable for static and dynamic strain measurements.**
- **They have very good fatigue properties.**
- **Stress relaxation and hysteresis is very less in these gauges.**

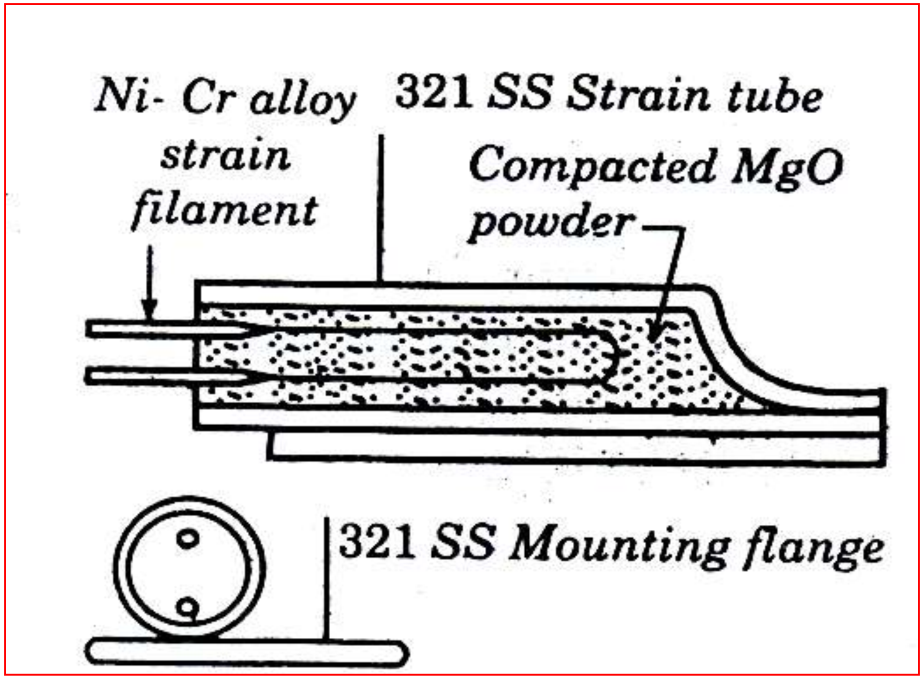
- **Weldable Strain gauges**
- Some of the limitations of the bonded type of metallic gauges are their comparatively costly, time consuming and complicated method of bonding.
- This realization led to the development of the weldable wire resistance strain gauge -a strain gauge capable of being installed in minutes and in any environment.

- This unique technique, utilizing capacitive discharge spot welding equipment eliminates the need for all bonding materials.
- The weldable strain gauge consists of a strain sensitive element, the Nickel Chrome or Platinum Tungsten, housed within a small diameter stainless steel tube.

- **The strain element is insulated from the tube with highly compacted ceramic insulation or metallic oxide powder, normally high purity magnesium oxide, which also serves as a strain transfer medium from the housing to strain element.**
- **These weldable gauges are equipped with a thin flange spot welded to the strain tube.**

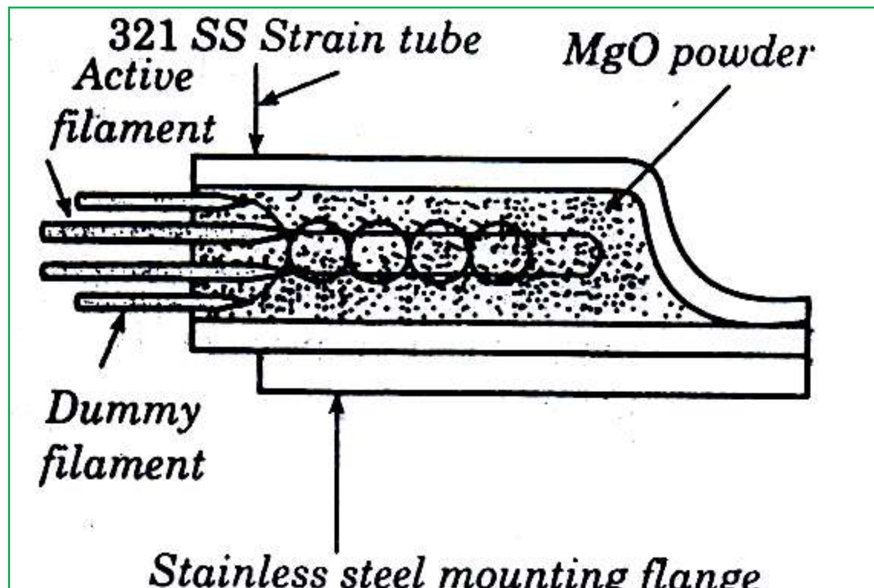
- This flange is subsequently spot welded to the structure under test and provides the bond required to transfer strain.
- Integral leads are attached to the basic gauge by welding.
- When the gauge is welded to a specimen and the test specimen put into tension or compression, the stress is transmitted through the weld to the mounting flange, into the strain tube, and through the magnesium oxide powder.

- The basic construction of a quarter-bridge or half-bridge, self-temperature compensated gauge is shown in Figs. and includes integral metal sheathed or flexible lead wire configurations.
- This gauge construction provides inherent mechanical and environmental protection for both the main filament and lead wires and is used over a broad temperature range from cryogenic to 65°C.



Half bridge gauge

Quarter bridge gauge



- **Weldable strain gauges can be used for a wide range of static and dynamic measurement applications.**
- **Their rugged construction and positive attachment make it possible to measure strain at higher or low temperatures and in server environments, including shock and vibration, steam, salt water, chemicals, and other corrosive atmospheres.**

- **Piezo-resistive strain gauges**
- **Crystals of silicon, germanium, quarts and Rochelle salt show a change in resistance when deformed by applying pressure.**
- **This effect can be utilized to measure strain.**
- **Such like gauges are called piezo-resistance strain gauges.**

- **Materials for Gauges**
- **A good gauge material should have the following qualities:**
- **High gauge factor**
- **High resistance**
- **Low temperature sensitivity**
- **High electrical stability**
- **High yield point stability**
- **High endurance limit**

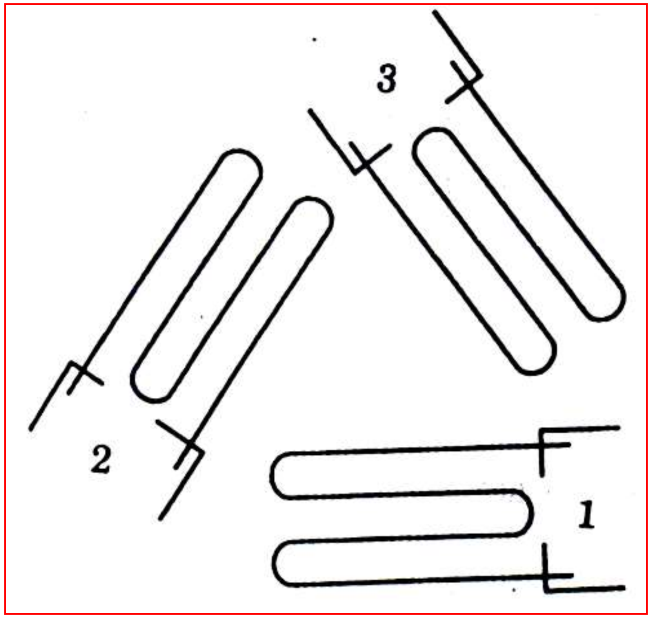
- Good workability
- Good solderability and workability
- Low hysteresis
- Good corrosion resistance
- Low thermal e.m.f. when joined with other metals.

Strain-Rosette Analysis

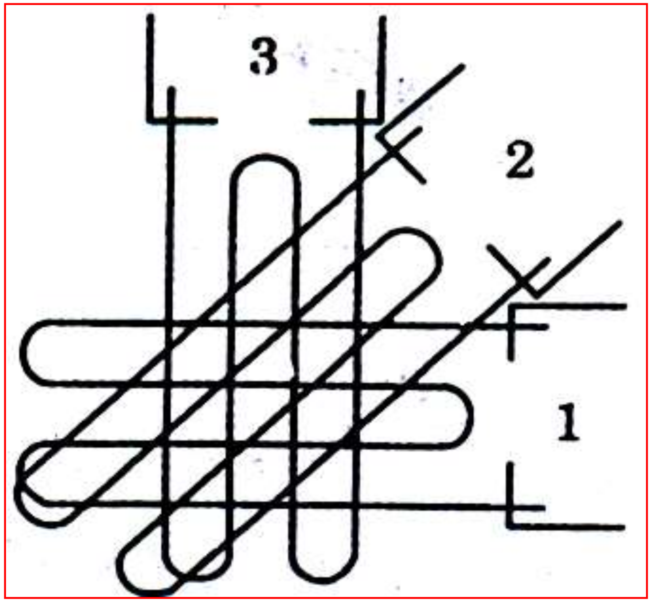
- **When the state of strain at a point and the direction of principal strains is known, then the strain gauges can be oriented along these directions, and strain measurements may be made.**
- **However, when the state of strain is not known, then three or more gauges may be used at the point to determine the state of strain at the point.**

- The resulting configuration is termed a strain rosette.
- Strain-rosette analysis is the art of arranging strain gauges as rosettes at a number of points on the object to be investigated, taking the measurements, and computing the state of stress at these points.

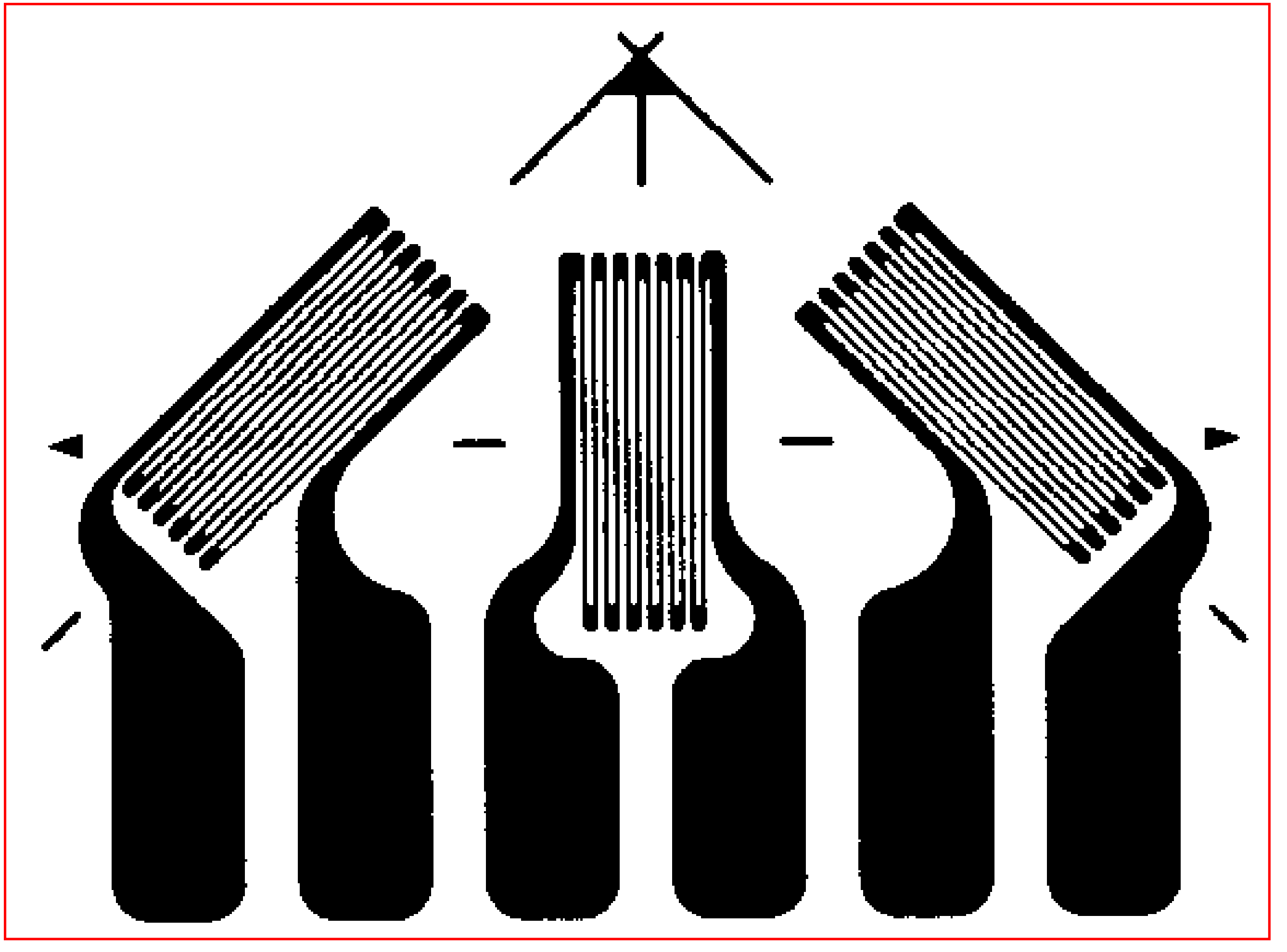
- **Strain rosette analysis is based on the assumptions of isotropic, homogeneous and linear material and of strain gradients so small that the strains can be considered as substantially uniform over the area covered by the rosette gauges.**



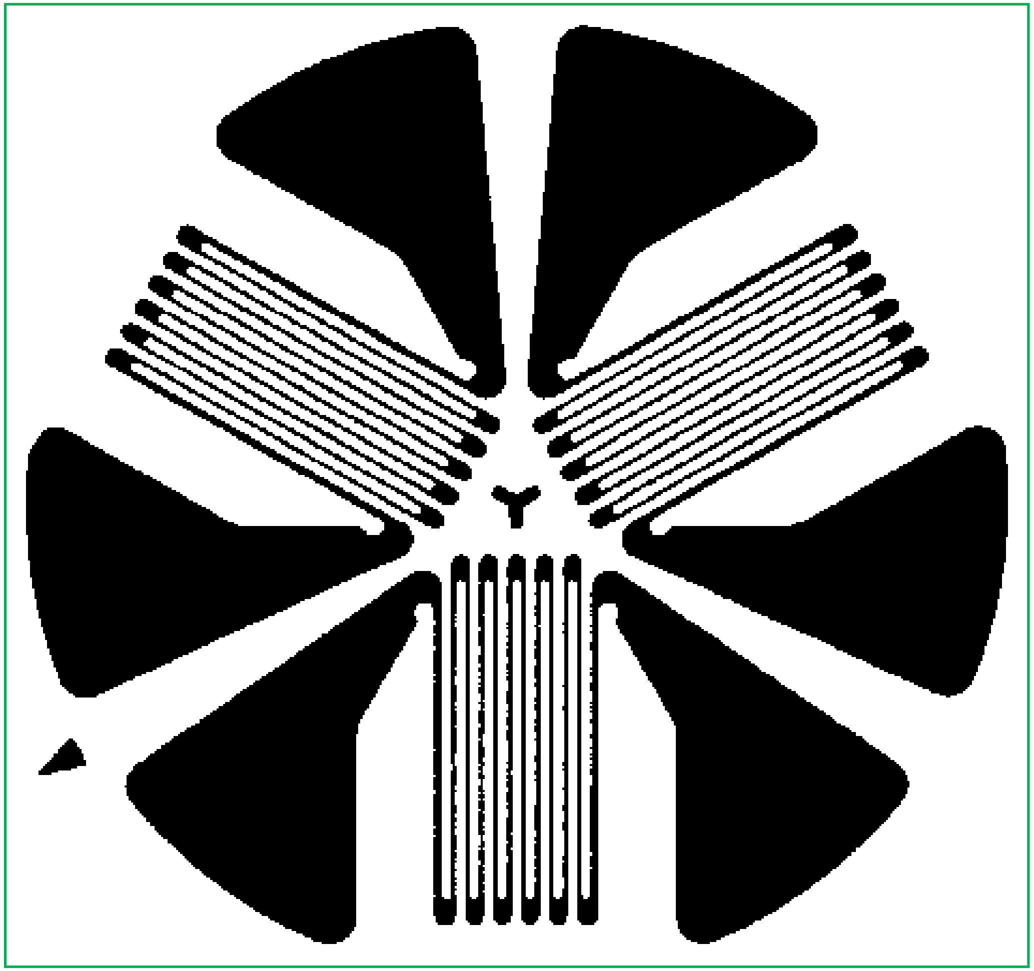
Delta rosette



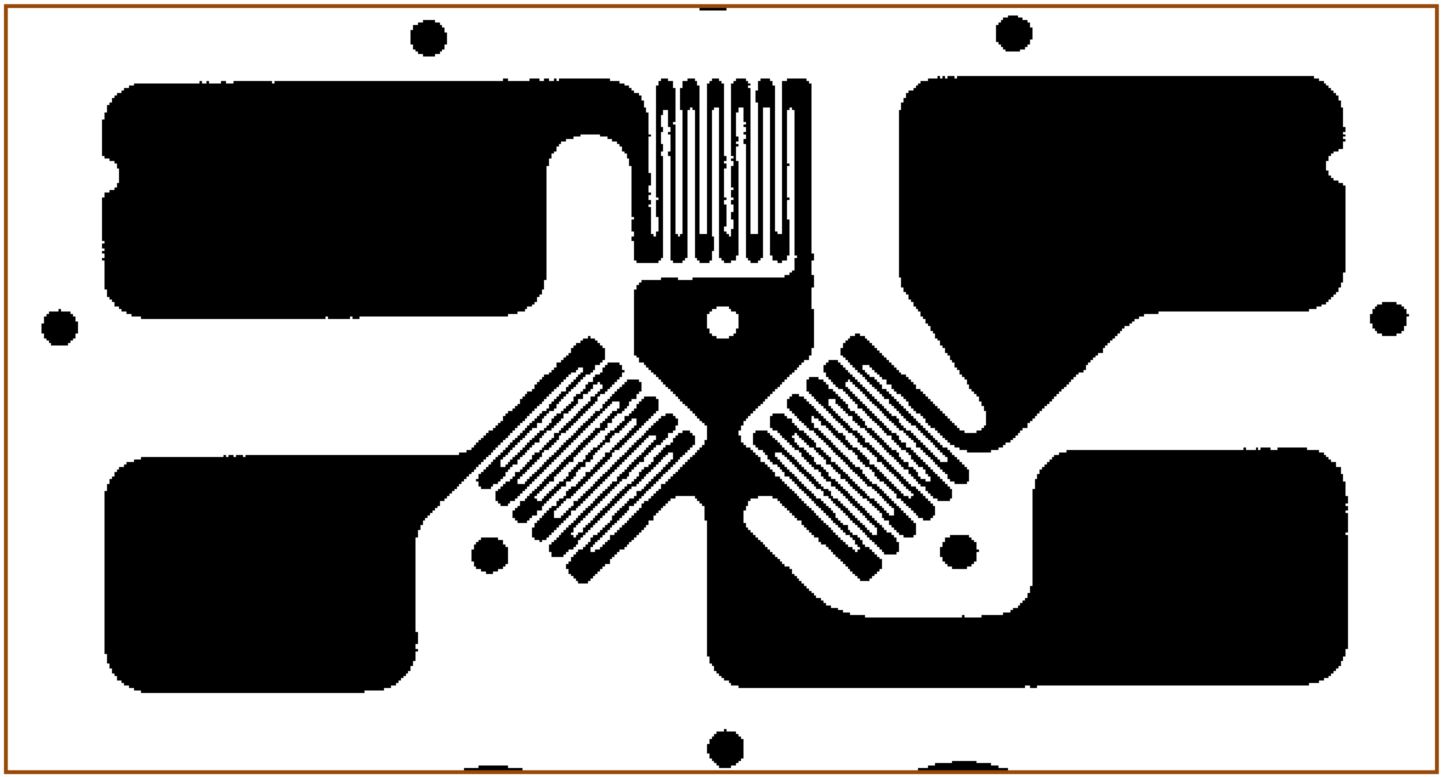
Three gauge rectangular rosette



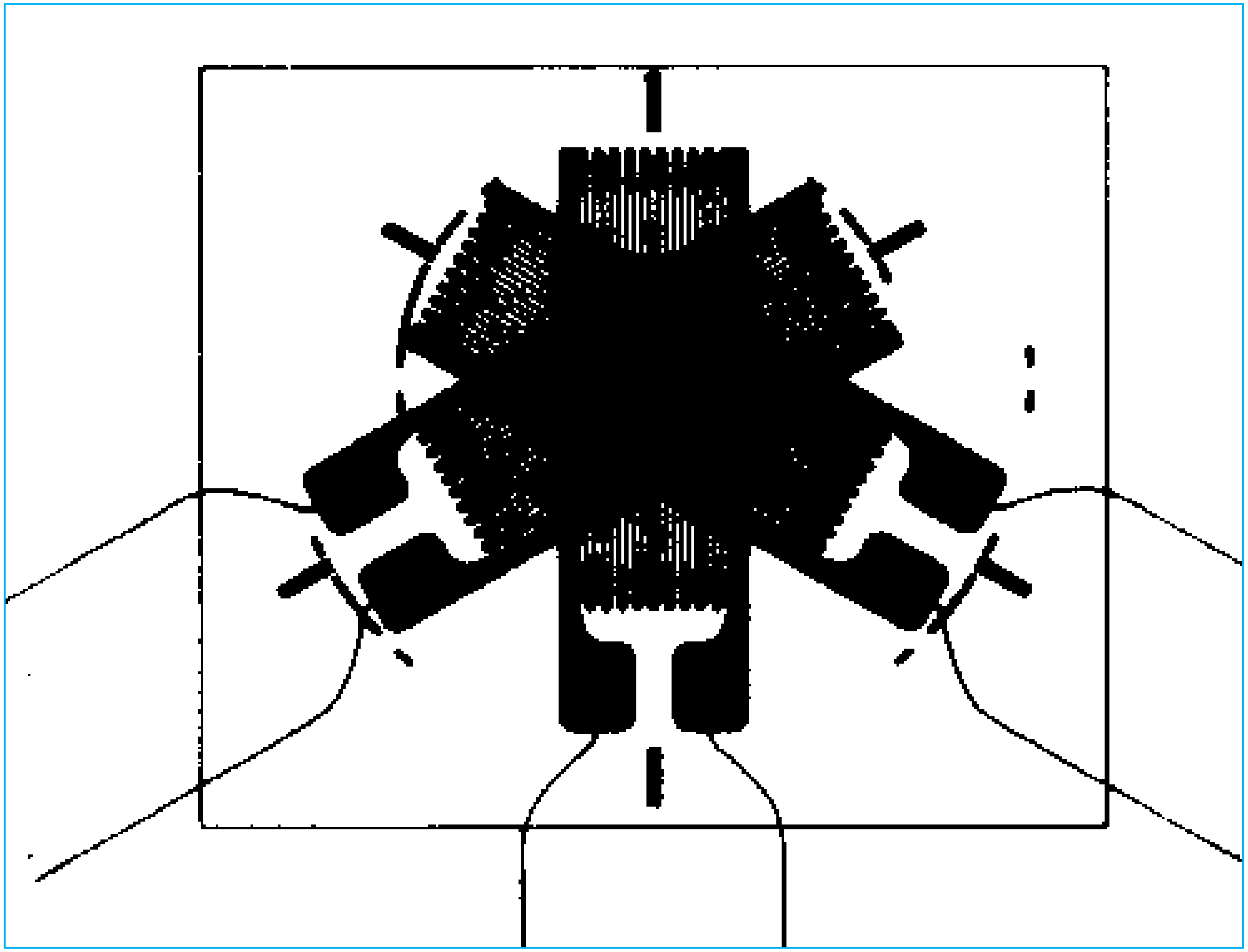
Rectangular Rosette



Delta Rosette



Delta Rosette



Stacked Delta Rosette

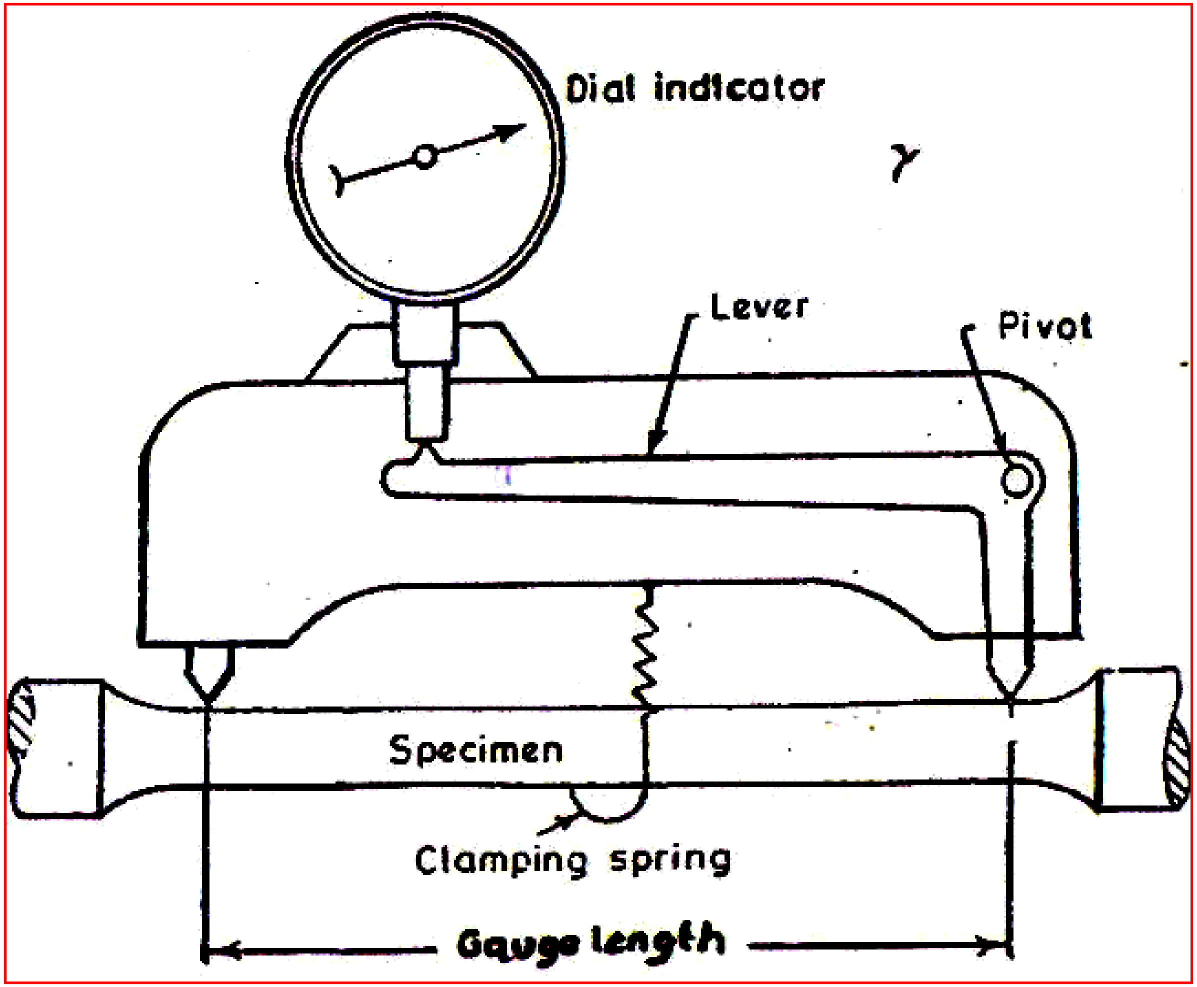
- The strain rosette analysis has the following advantages:
- Extreme simplicity and speed of application.
- Possibility of allowing for transverse effects.
- No requirements for additional equipment.
- The possibility of training relatively unskilled persons to use the method.

- Basic Circuits (Constant Voltage Type)
- There are two types of circuits used for strain measurements.
- The Wheatstone bridge
- (a) Null balance type
- (b) Out-of-balance type
- Potentiometer

Mechanical Strain Gauges

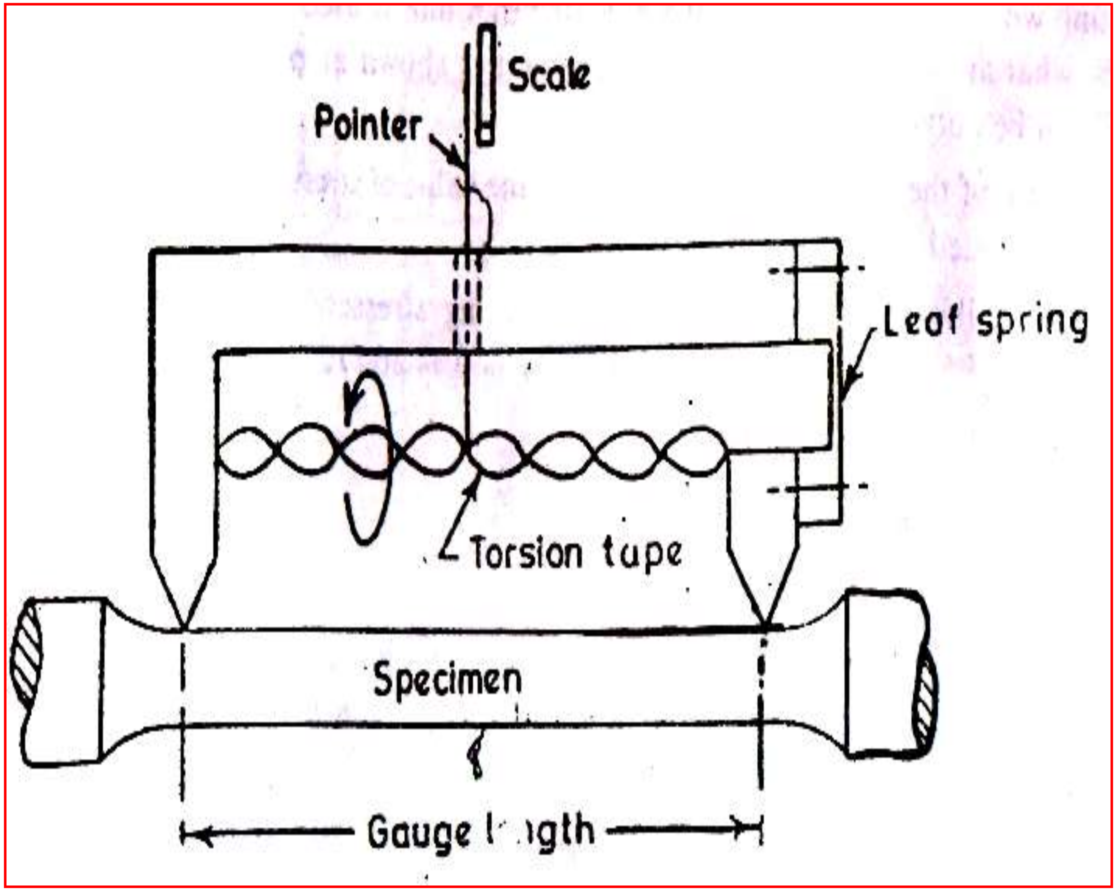
- Mechanical devices are generally known as extensometers and are used to measure strain under static or gradually varying loading conditions.
- An extensometer is usually provided with two knife edges which are clamped firmly in contact with the test component at a specific distance or gauge length apart.

- When the test component is strained, the two knife edges undergo a small relative displacement.
- This is amplified through a mechanical linkage and the magnified displacement or strain is displayed on a calibrated scale.



Berry Strain Gauge

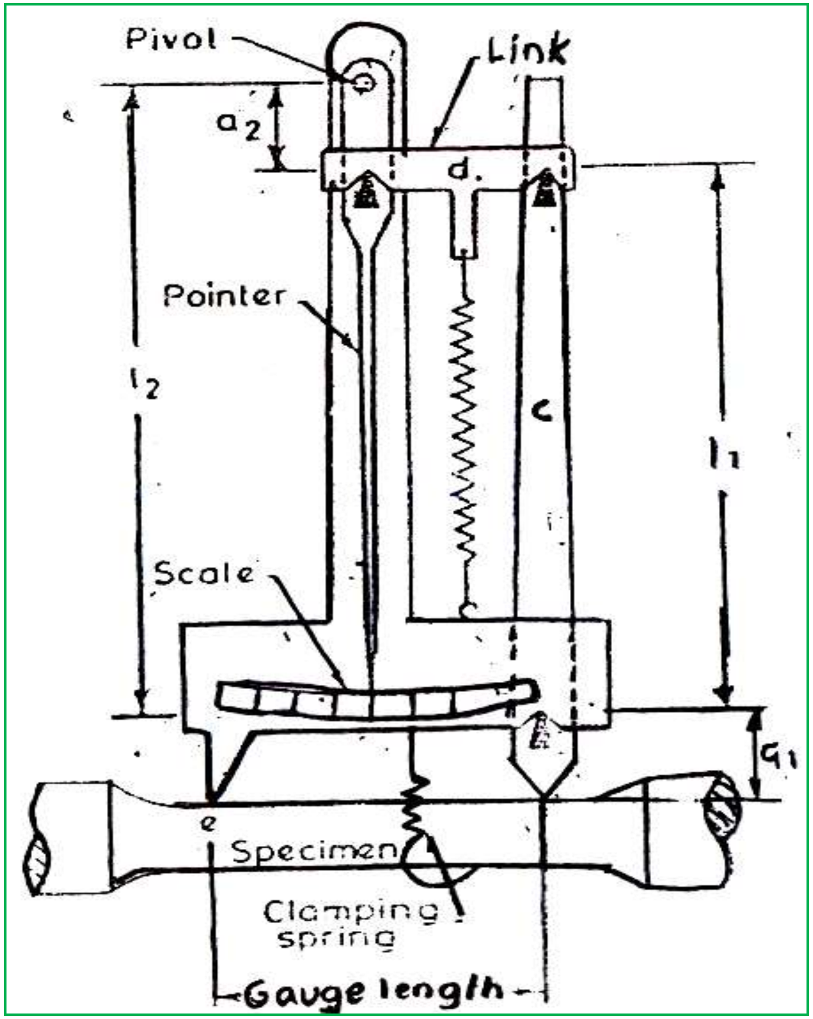
- The Berry strain gauge uses a system of a lever and dial gauge to magnify the small displacement between the knife edges.
- It can Measure strains down to 10 micro strain over a 50 mm gauge length.



Johansson Extensometer

- In the CEJ extensometer is a twisted metal strip or torsion tape stretched between the knife edges.
- Half the length of this strip is twisted in one direction while the other half is twisted in the opposite direction.
- A pointer is attached at the centre.
- The displacement of the knife edges,

- i.e. starching (stiffen) of the torsion tape is converted into a highly amplified rotational movement of the pointer.
- The CEJ extensometer can measure strain with a sensitivity of 5 micro strain over a gauge length of 50 mm.



Huggenberger Extensometer

- **In the Huggenberger extensometer a set of compound levers is used to magnify the displacement of the knife edges.**
- **The extensometer is highly accurate, reliable, light-weight and self-contained.**
- **The movable knife edge *(f)* rotates the lever *c* about the lower pivot.**

- The lever *c* in turn rotates the pointer through the link *d*.
- The magnification ratio is given by $l_1 l_2 a_1 a_2$.
- Extensometers with this ratio varying between 300 and 2000 and with gauge lengths in the range 6.5 to 100 mm are available.

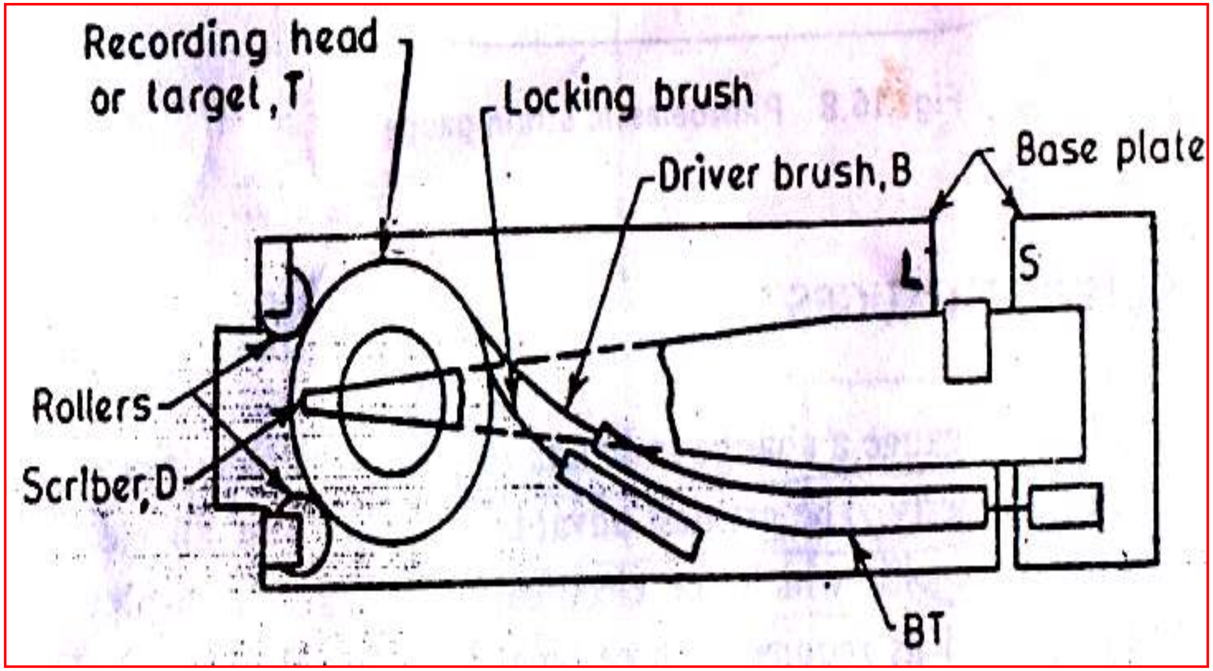
- **The sensitivity of these extensometers could be as high as 10 micro strain.**
- **It is well suited for applications where its unusually large height does not pose problems of instability in mounting.**

Scratch Gauge

- **The scratch gauge is a self-contained compact device providing a permanent record of displacement over a period of time.**
- **In this gauge the relative displacement between two stainless steel base plates *L* and *S* secured to the test component causes a scriber *D***

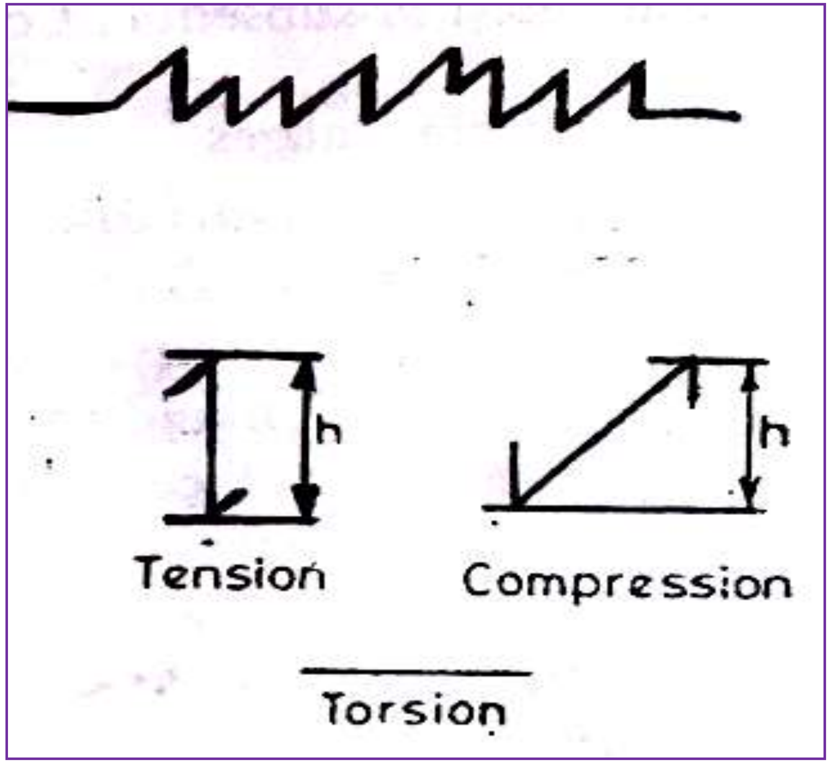
- to scratch sharply the actual component deformation on a small brass (target, *T*).
- The target is held in position by two tiny rollers and two stainless steel brushes.
- The free end of the long driver brush *B* engages a peripheral groove of the target.
- It is also guided in a bent tube *BT.*

- When a tensile deformation is removed or a compressive deformation is produced, the plates *L* and *S* move towards each other.
- This causes the driver brush *B* to rotate the circular target by a small amount.
- However during a tensile deformation the driver brush *B* just slides back in the target groove without rotating it.



Scratch gauge (Prewitt Associates, USA)

- **Thus tensile movements scribe a line parallel to the gauge axis.**
- **Compressive movements and removal of tensile strain scribe a line at approximately 45^{o} to the gauge axis.**
- **The height *h* of the recorded data is the product of the strain and gauge length.**



Scratch gauge record

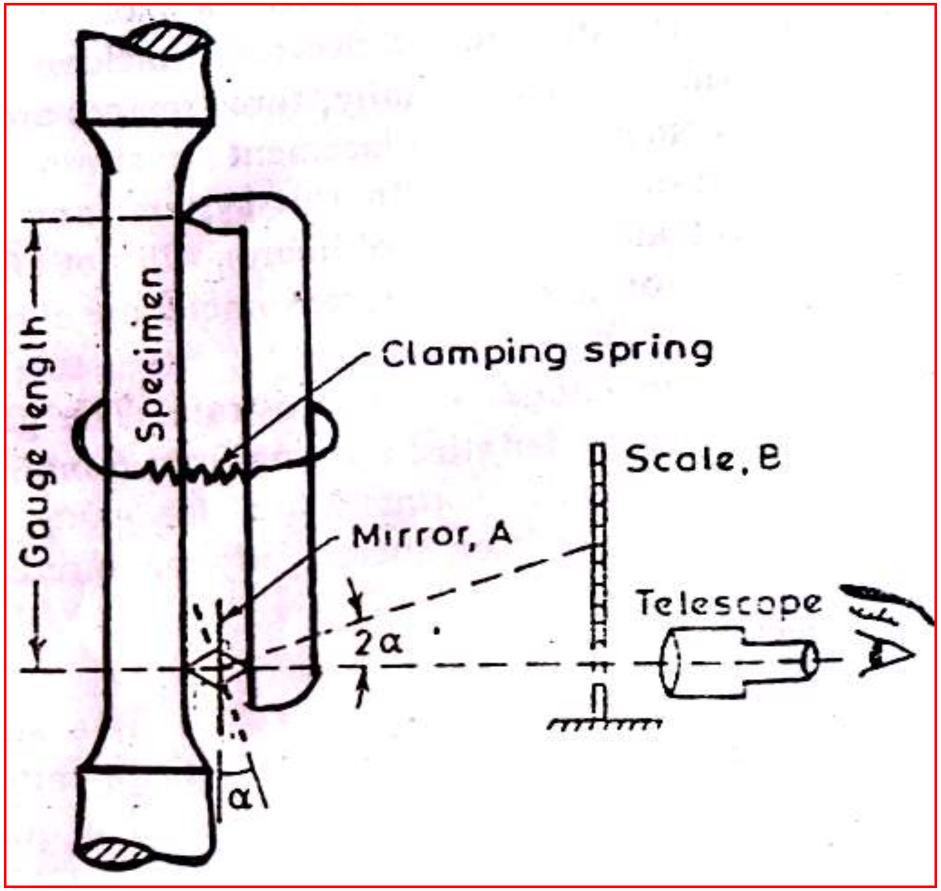
- **The traces on the target are evaluated by viewing them with a microscope having a calibrated eye-piece scale.**
- **The minimum strain that a scratch gauge can sense is about 100 micro strain.**
- **The gauge lengths of these gauges are rather large.**

- The scratch gauge is compact in size and weighs less than 30 g.
- It can be attached to almost any surface with clamps or screws or adhesive bonding.
- It can measure stresses under all types of loading-static, fatigue or shock.
- It can be used to record stresses in all types of environments - room and elevated temperatures, under water, under radiation, etc.

Optical Strain Gauges

- In mechanical-optical gauges a combination of mechanical and optical levers are used to amplify the relative displacement between the knife edges.
- The moving knife is pivoted.
- So that it rotates while undergoing displacement.

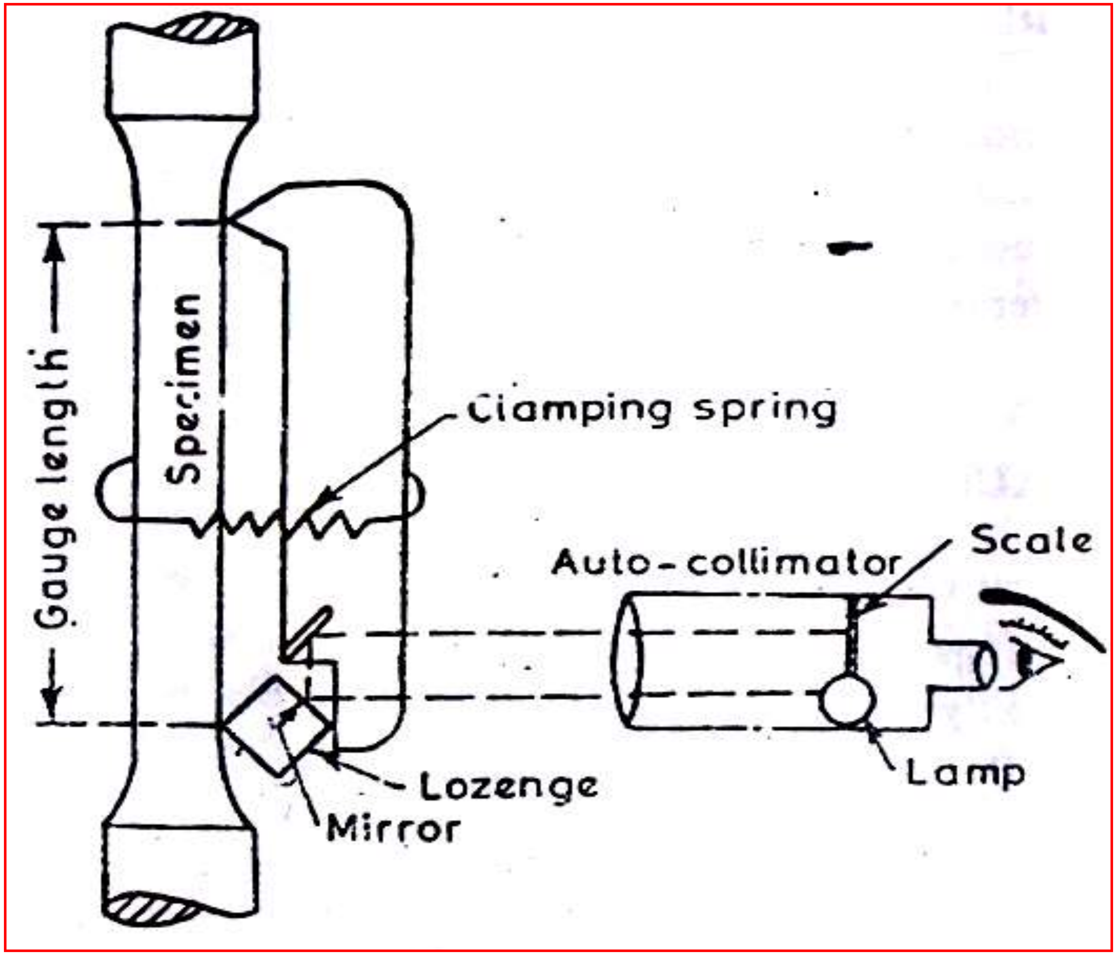
- **The principle of the signal mirror system is illustrated in Fig.**
- **The pivoting knife edge carries a mirror *A*.**
- **The reflection of an illuminated scale *B* in this mirror is viewed through the observing telescope.**



Martens optical gauge

- **Any deformation of the structure to which this gauge is fixed, rotates the mirror *A* and thereby brings different portion of the scale into view.**
- **Thus the change in the reading on the scale is directly proportional to the deformation being measured.**

- A schematic diagram of the Tuckerman optical gauge and the autocollimator used with it is given in Fig.
- The autocollimator carries both the source of a parallel beam of light and an optical system with reticule to measure the deflection of the reflected ray.
- A tungsten-carbide, Rocker (lozenge) functions as the moving knife edge.



Tuckerman optical gauge (American Instrument Co., Inc)

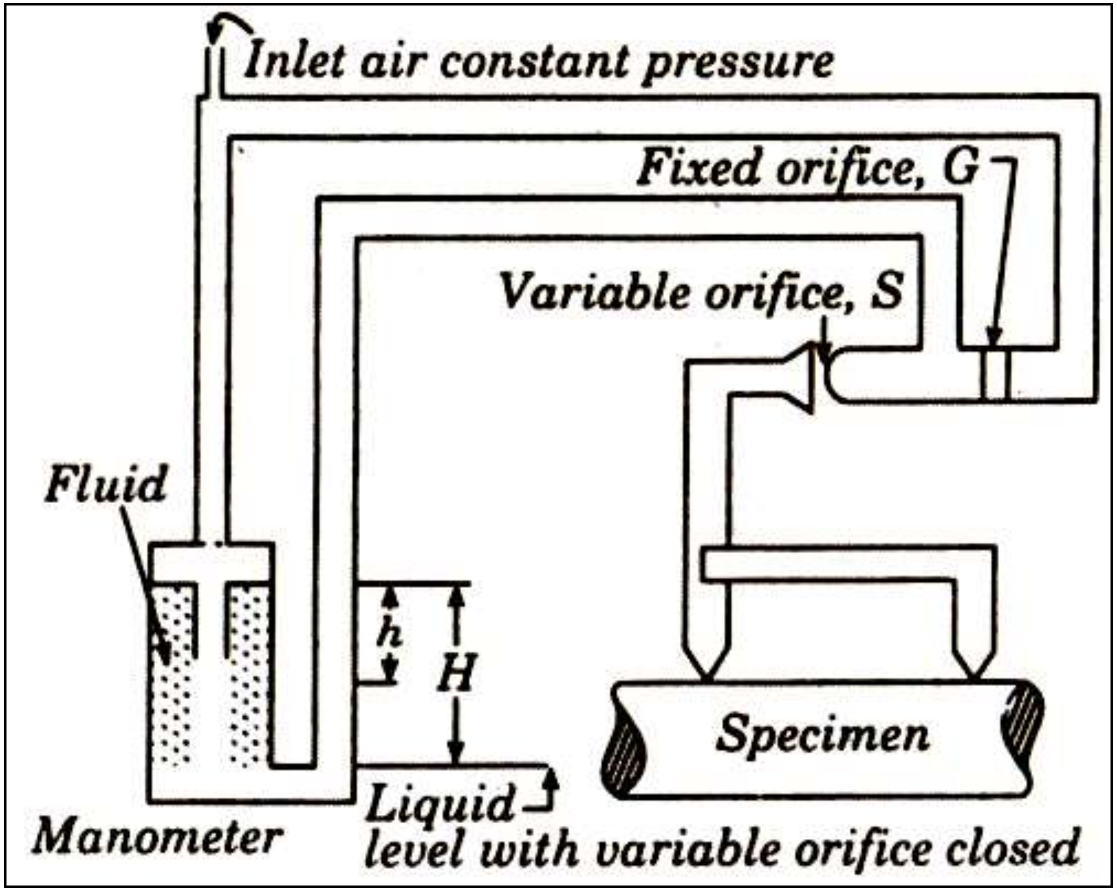
- **One face of this lozenge is polished to function as a mirror.**
- **The rotation of the lozenge resulting from a deformation of the structure deflects the incident parallel light beam back to the measuring reticule.**
- **Actually, three images are visible on the reticule-**

- One giving the measured displacement or strain and the other two helping the alignment of the gauge.
- In this, any relative motion between the component and the autocollimator will not affect the measurement.
- Errors due to rotation of the extensometer are eliminated in this system.

- **The sensitivity of the Tuckerman gauge is 2 micro strain.**
- **The gauge is available with a wide range of gauge lengths, starting from 6 mm.**
- **It can reliably measure both static and dynamic strains.**
- **With the gauge, cyclic strains up to a frequency of 180 c/s have been successfully measured.**

Pneumatic Strain Gauges

- **The principle of operation of an air or pneumatic gauge depends upon the relative discharge of air between a fixed orifice and a variable orifice.**
- **Fig. shows a pneumatic gauge.**
- **Air under constant pressure H, flows through two orifices placed in series.**
- **The pressure *h* which prevails between these two orifices is a function of the ratio of their areas.**



Pneumatic Gauge

- The fixed orifice G is called the nozzle and the second orifice S, which is smaller, is called the exhaust orifice and is of variable area of cross-section.
- As a result of it, the pressure h serves to measure the dimension of S.
- Air after passing through the orifice G, strikes the top plate and is vented to the atmosphere.

- **The flow of air through the two orifices in series must be equal if incompressibility is assumed.**
- **This assumption is practically valid as the pressures are quite low.**
- **Let,**

A_G = cross sectional are of nozzle orifice G

A_S = cross sectional area of discharge orifice S

C_G, C_S = coefficients of contraction for the orifices

ρ = density of air

g = acceleration due to gravity

Since the flow through each orifice is the same, hence

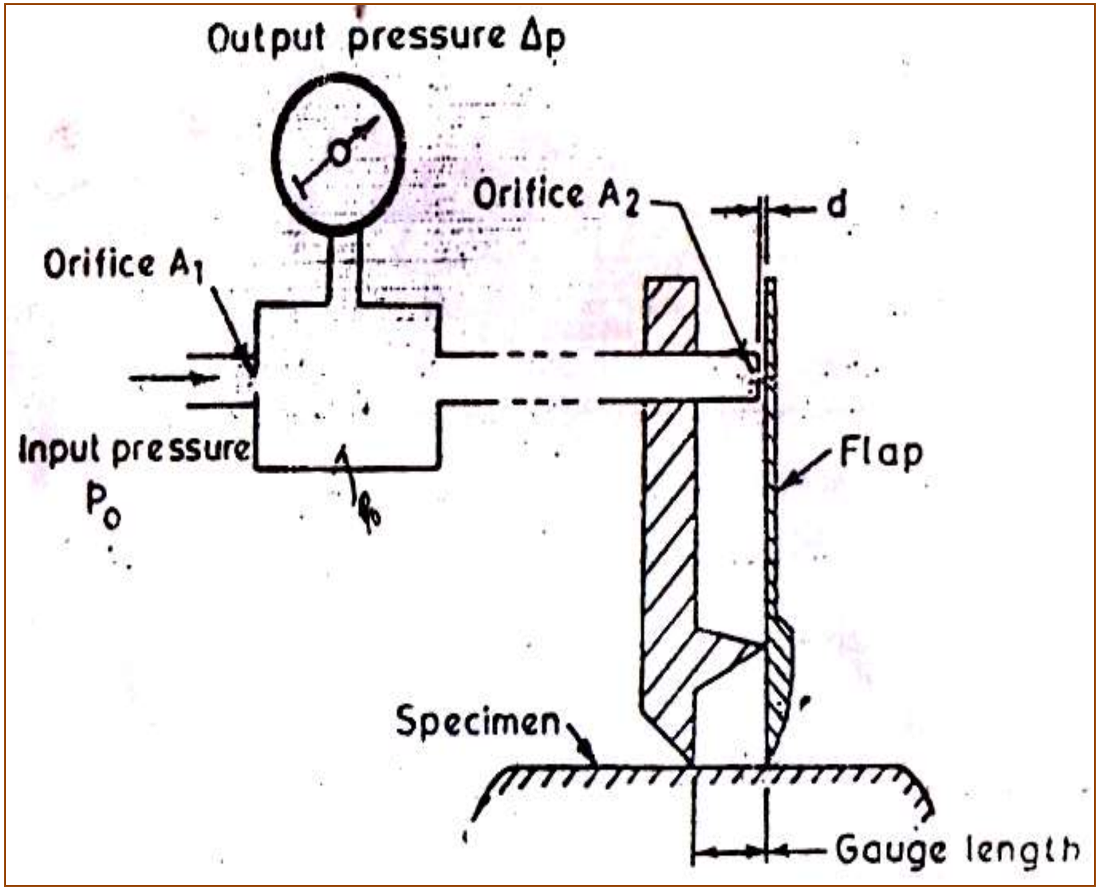
$$C_G, A_G \sqrt{\frac{2g(H-h)}{\rho}} = Cs\ \mathrm{As}\ \sqrt{\frac{2\mathrm{gh}}{\rho}}$$

When $C_S = C_G = C$,

$$h = \frac{H}{1+\left(A_S / A_G\right)^2}$$

- **When the specimen is leaded, the distance between the two gauge points changes.**
- **This elongation is transmitted through the level’s system to the pneumatic gauge, where it changes the gap between orifice S and the top plate, this changing the area As in direct proportion to the strain.**

- From Eq. it is obvious that the manometer reading varies as a quadratic function of the strain.
- However, it has an inflection point when $h/H = 3/4$ or $A_S/A_G = 0.58$.
- Hence, for values in this neighbor-hood, the relation is very nearly linear.
- Multiplication factors of 100 are possible with this type of pneumatic amplification.



Pneumatic strain gauge – Single pressure output

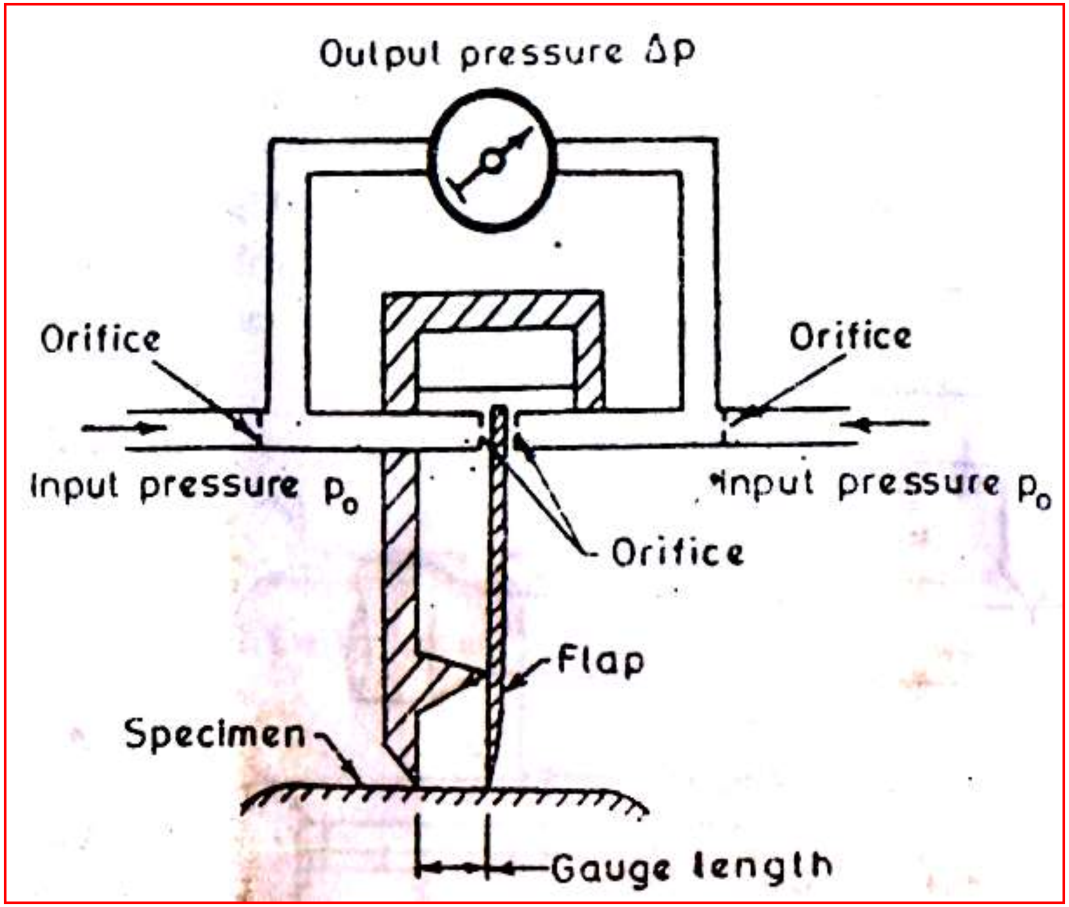
- Figure shows the basic arrangement in a pneumatic strain gauge.
- Air at constant pressure flows through two orifices of cross – sectional areas A_1 and A_2.
- The area A_2 of the variable area orifice is a function of the gap d which varies as the distance between the knife edge changes.

- The pressure Δp built up in the chamber is approximately given by

$$\boxed{\Delta p = \frac{p_0}{1+\left(A_2 / A_1\right)^2}}$$

- Thus the relationship between Δp and the displacement of the extensometer d is nonlinear.

- **However, with proper design this non linear characteristics of the gauge can be minimized and a nearly linear characteristic can be obtained over a narrow range of displacement.**
- **Better linearity can be obtained in the arrangement shown in figure.**



Pneumatic strain gauge – Differential pressure output

- Magnifications up to 100,000 and gauge lengths as small as 1 mm are possible to achieve in these gauges.
- Pneumatic gauges are sensitive, robust and reliable.
- They are suitable for both static and dynamic strain measurements.

Acoustical Strain Gauge

- **The vibrating wire or acoustical gauge consists essentially of a steel wire tensioned between two supports a predetermined distance apart.**
- **Variation of the distance alters the natural frequency of vibration of the wire and this change in frequency may be correlated with the change in strains causing it.**

- An electro-magnet adjacent to the wire may be used to set the wire in vibration and this wire movement will then generate an oscillating electrical signal.
- The signal may be compared with the pitch of an adjustable standard wire, the degree of adjustment necessary to match the two signal frequencies being provided by a tensioning screw on the standard wire.

- Calibration of this screw allows a direct determination of the change of length of a measuring gauge to be made once the standard gauge has been tuned to match the frequency of the measuring wire.
- Tuning is now more usually accomplished by feeding the two signals into the two pairs of plates of an oscillograph.

- Matching of the tones is simplified and made more accurate by tuning out the beats which results when the vibration frequencies of two wires are nearly the same, which can be compared by using earphones.
- The fundamental frequency of a stretched wire may be estimated from the expression:

$$\boxed{f = \frac{1}{2L}\sqrt{\frac{P}{m}} = \frac{1}{2L}\sqrt{\frac{(E\delta L)/L}{m}A}}$$

Where, A = cross sectional area of vibrating wire

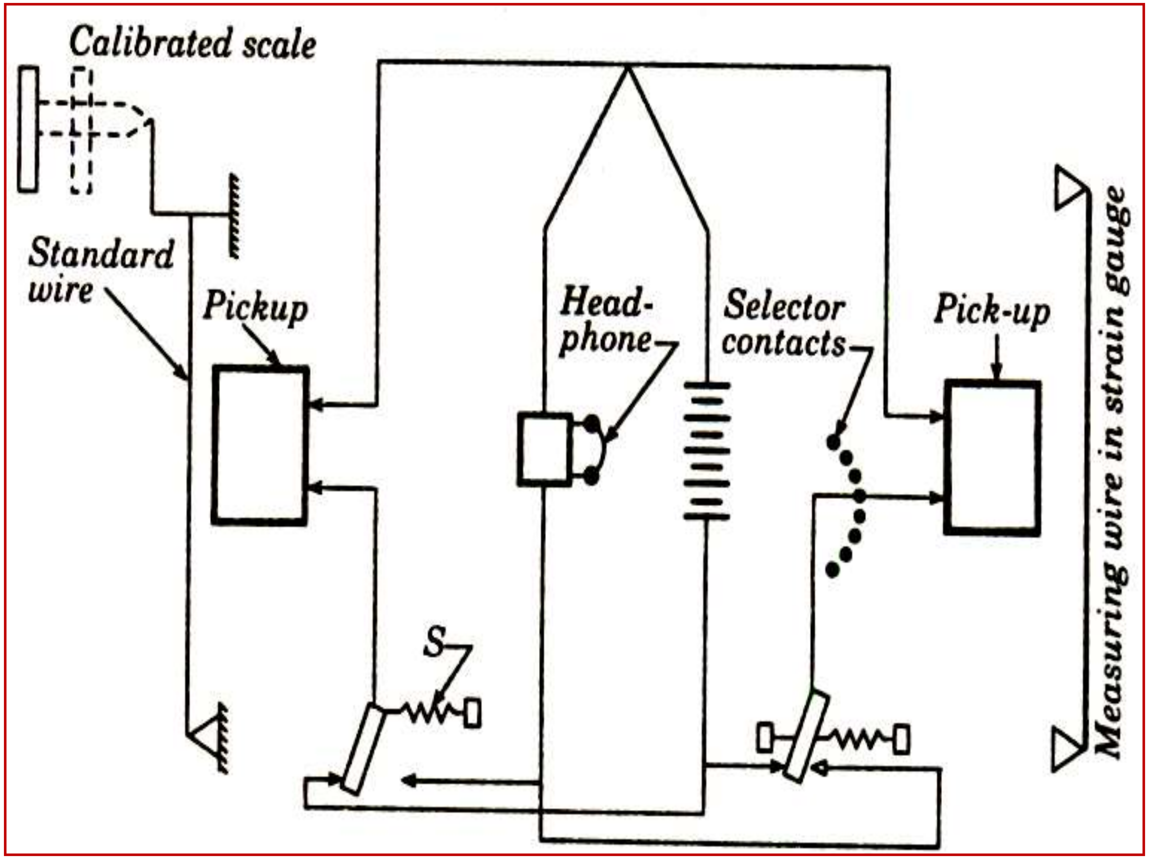
E = Young's modulus of wire material

L = length of vibrating wire

m = mass per unit length of the wire

P = tensioning force in the wire

δL = increment in length of the vibrating wire.



- **Figure shows an acoustical gauge developed by Dr. O. Schaefer about 1933.**

- The sensitivity of this gauge is very high, with possible determinations of displacement of the order of 0.25 μ cm.
- The range is limited to about 1/1000 of the wire length.
- The gauge is temperature sensitive unless the thermal coefficients of expansion of the base and wire are closely matched over the temperature range encountered during a test.

Electrical Strain Gauges

- **An electrical stain gauge is a device in which a change in length produces a change in some electrical characteristics of the gauge.**
- **The electrical strain gauges may be classified as follows:**
- **The inductance or magnetic strain gauges.**
- **The capacitance strain gauges.**
- **The electrical resistance strain gauges.**

Inductance Strain Gauges

- **The inductance type of strain gauges in which the strain is measured as a change in the magnetic field, was developed as a strain gauge about 1930 by Shamberger.**
- **Since then this gauge has been used for various applications, particularly in motion measurements.**

- **An important application of inductance type of gauge is the linear variable differential transformer, developed by Schaevitz about 1947.**
- **An electric inductance gauge is a device in which the mechanical quantity to be measurement produces a change in the magnetic field, and hence in the impedance, of a current – carrying coil.**

- The impedance of a coil depends on its inductance and on its effective resistance, and either or both of these quantities can be made sensitive to the mechanical quantity being measured.
- The inductance which is changed can be either the self inductance of the coil or its mutual inductance with respect to another coil.

- Depending upon the method of varying the impedance, electric – inductance gauges may be classified as follows:
- **Variable – air – gap gauges:** In which the reluctance of the magnetic field is varied by changing the air gap.
- **Movable – core solenoid gauges:** In which the reluctance of the magnetic circuit is varied by changing the position of the iron core in the coil.

- **Eddy current gauges:** In which the losses in the magnetic circuit are varied by changing the thickness or position of the high – loss element inserted in the magnetic field.
- **Magnetostriction gauges:** In which the reluctance of the magnetic circuit is varied by changing the stress in the magnetic core of the coil.

- **The impedance of a coil to the passage of alternating current is given by the expression:**

$$\boxed{Z = \sqrt{(2\pi fL)^2 + R^2}}$$

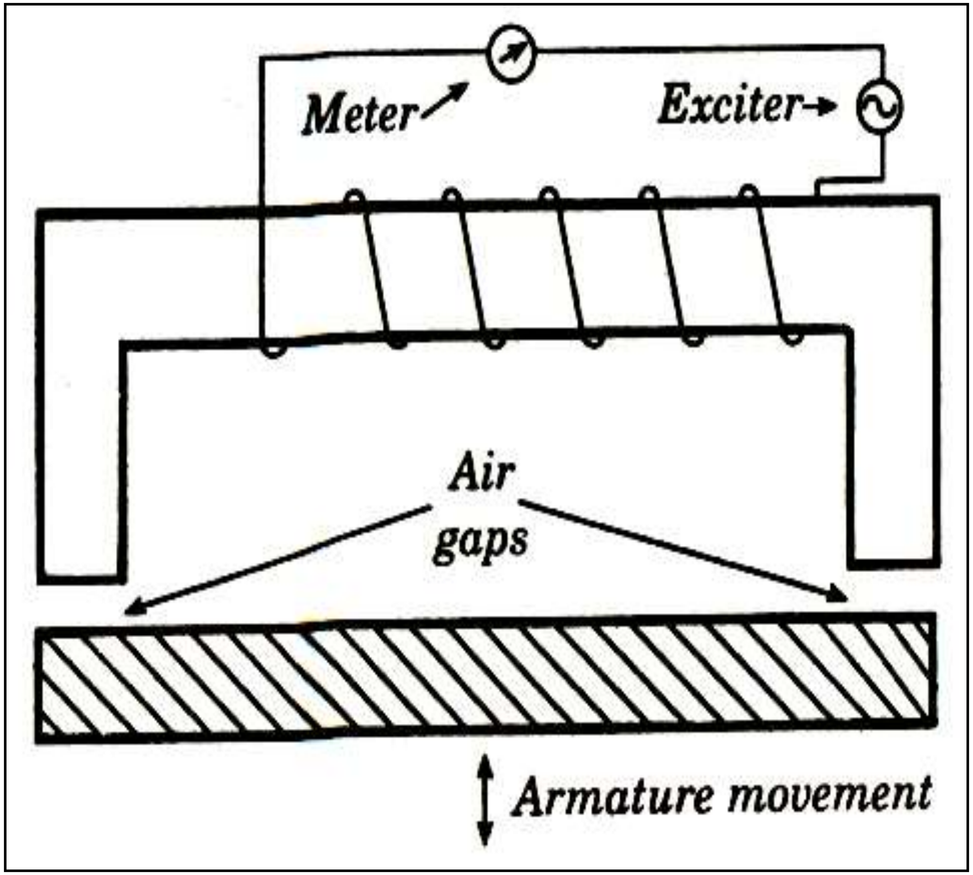
Where,

Z = impedance in ohms

f = frequency in hertzs

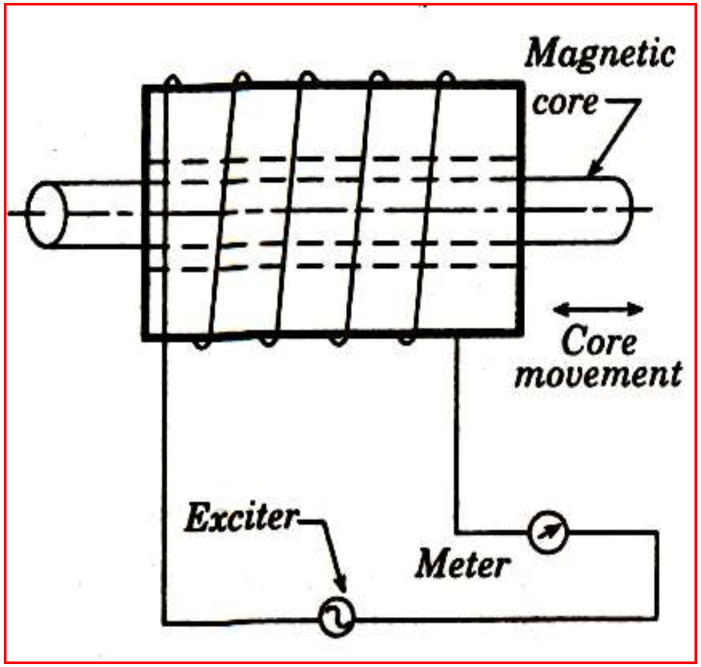
L = inductance of the coil in henrys

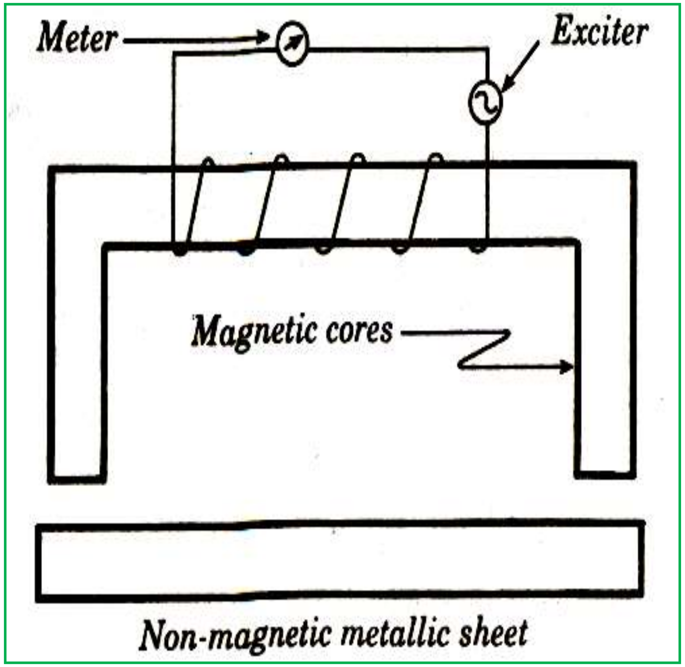
R = resistance component in ohms.



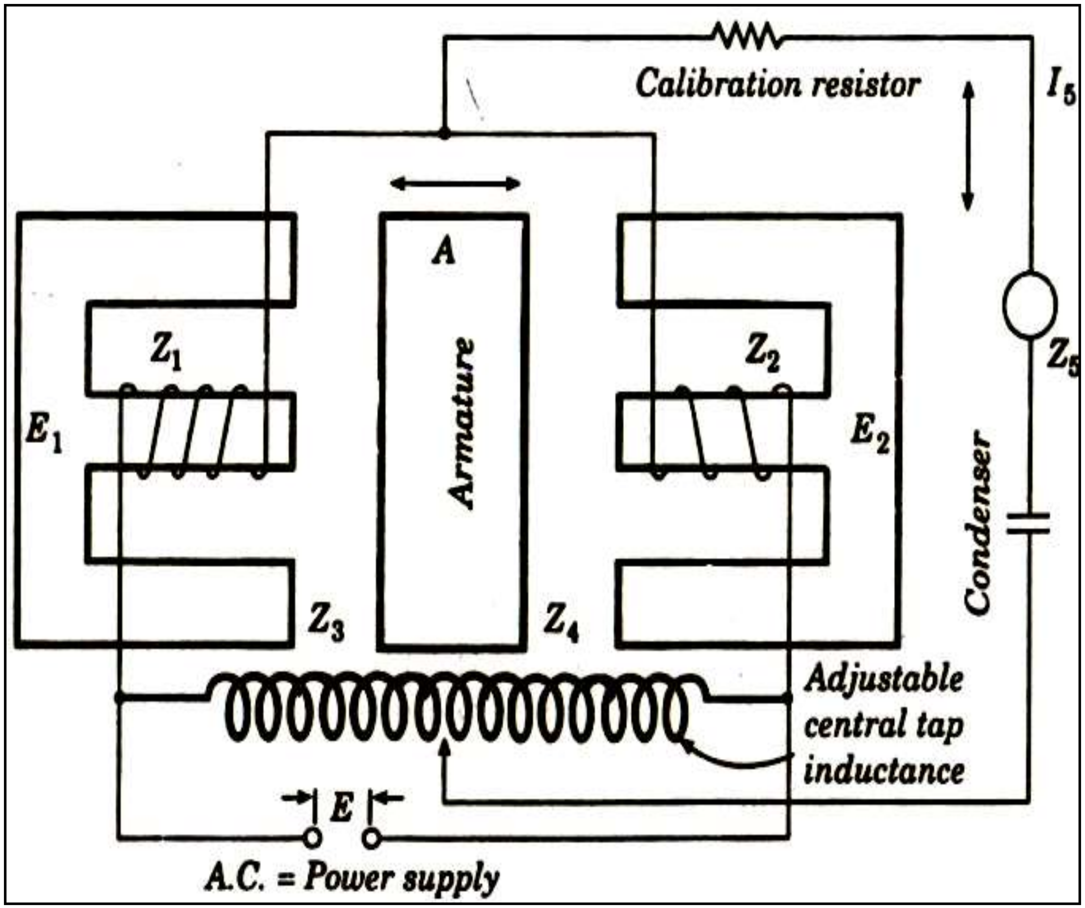
Variable air gap gauge

- **Moving coil solenoid gauge**
- **Eddy current gauge**

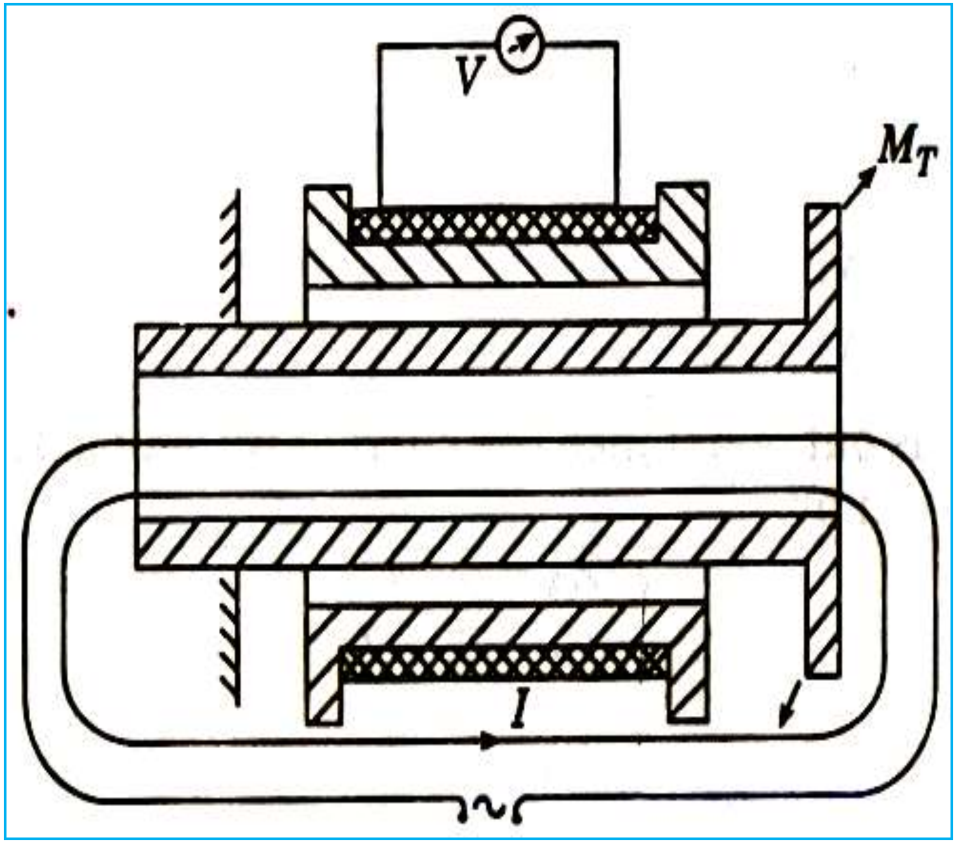




- **Basic impedance bridge circuit**



- **Electromagnetic strain gauge (Wiedemann's effect)**



Capacitance Strain Gauge

- The electrical capacity between parallel plates is given by:

$$\boxed{C = \frac{8.86 \times 10^{-3} KA(N-1)}{h}}$$

Where

C = capacitance of plates

K = dielectric constant of the medium between the two plates

N = number of plates

h = distance between plates, mm

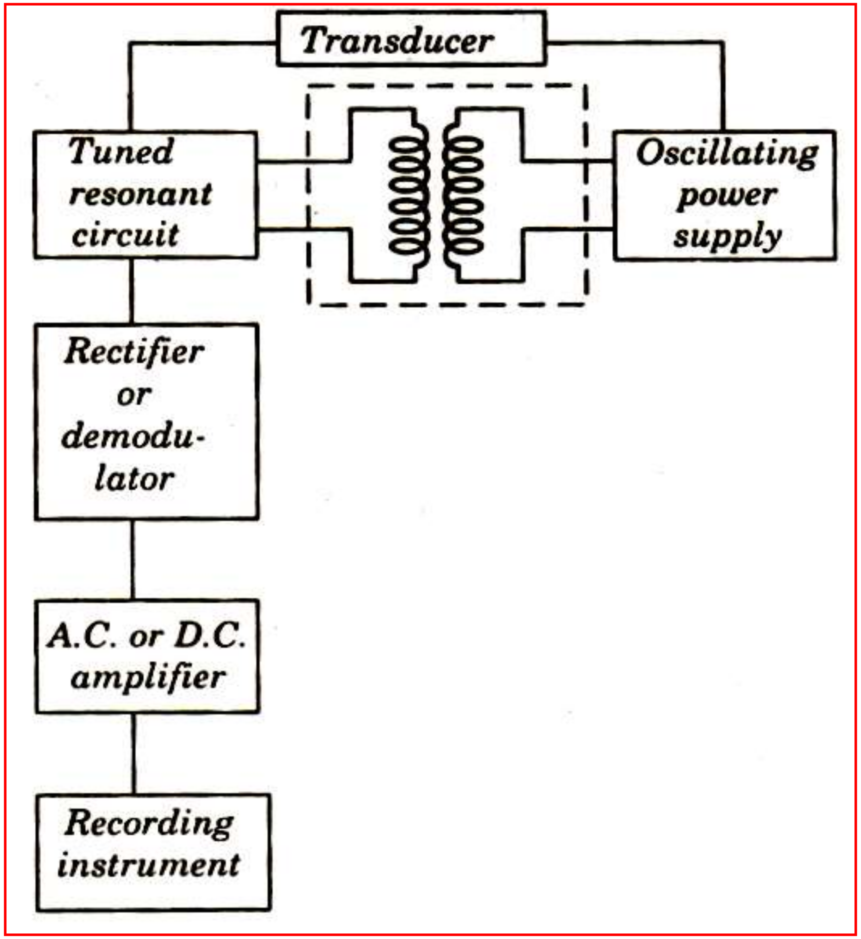
$$\boxed{\frac{C}{dh} = \frac{8.86\times10^{-3}KA(N-1)}{h^2} = -\frac{C}{h}}$$

$$\boxed{\varepsilon = \frac{\Delta h}{l_0}}$$

$$\boxed{\varepsilon = \frac{h}{l_0}\ \frac{\Delta C}{C}}$$

- **Where l_0 is the gauge length**

- Simplified diagram of a capacitance transducer circuit



- Two types of circuits may be used to measure the change in capacitance of a gauge.
- The first method employs an amplitude – modulated signal, while the second method uses a frequency – modulated signal.
- Dielectric, mounting and clamping difficulties make this gauge not too desirable.

- Capacitance gauges are small in size and they have excellent high – frequency response and high temperature resistance, as well as good linearity resolution and ability to measure both static and dynamic quantities.
- These gauges are sensitive to temperature, vibrations, have high impedance output and complexity of associated electronic equipment.

STRAIN MEASUREMENT

EXTENSOMETERS

- **Strain gauges are mostly used to measure strains on the free surface of a body.**
- **The state of strain at any point on the free surface of a body can be characterized in terms of three Cartesian strain components $\in_{xx}$, $\in_{yy}$ and γ_{xy}.**

$$\in_{xx} = \frac{\partial u}{\partial x} \in xy = \frac{\partial v}{\partial y}$$

$$\gamma_{XY} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Where *u* and *v* are the displacement components in *x* and *y* directions respectively.

- These equations suggest that if the two displacements u and v can be measured at all points on the surface of a body, strains at any point on the surface can be determined.
- It is seen from the Eq. that the Cartesian strains are actually the slopes of the displacement surfaces u and v.

- **For precision in the estimation of the slopes of the displacement surfaces, the in-plane displacements *u* and *v* should be determined quite accurately.**
- **However, particularly for small elastic strains, the in-plane displacements are exceedingly small.**

- **No versatile easy method is yet available for the direct measurement of these displacements.**
- **This difficulty is overcome partially by using a strain gauge to measure the change in the distance between two points on the surface of the body due to straining.**

- **This change in length is converted to axial strain by the following relationship.**

$$\in_{xx} = \frac{\Delta u}{\Delta x}$$

- **Here Δ *u* is the change in length over a distance or the gauge length, Δx.**

- Strain gauges of all types are essentially devices that sense the change in length, magnify it and indicate it in some form.
- They can be classified into broadly five groups on the basis of the physical employed for the magnification of change in length.

- »Mechanical
- »Optical
- »Electrical
- »Pneumatic, and
- »Acoustical

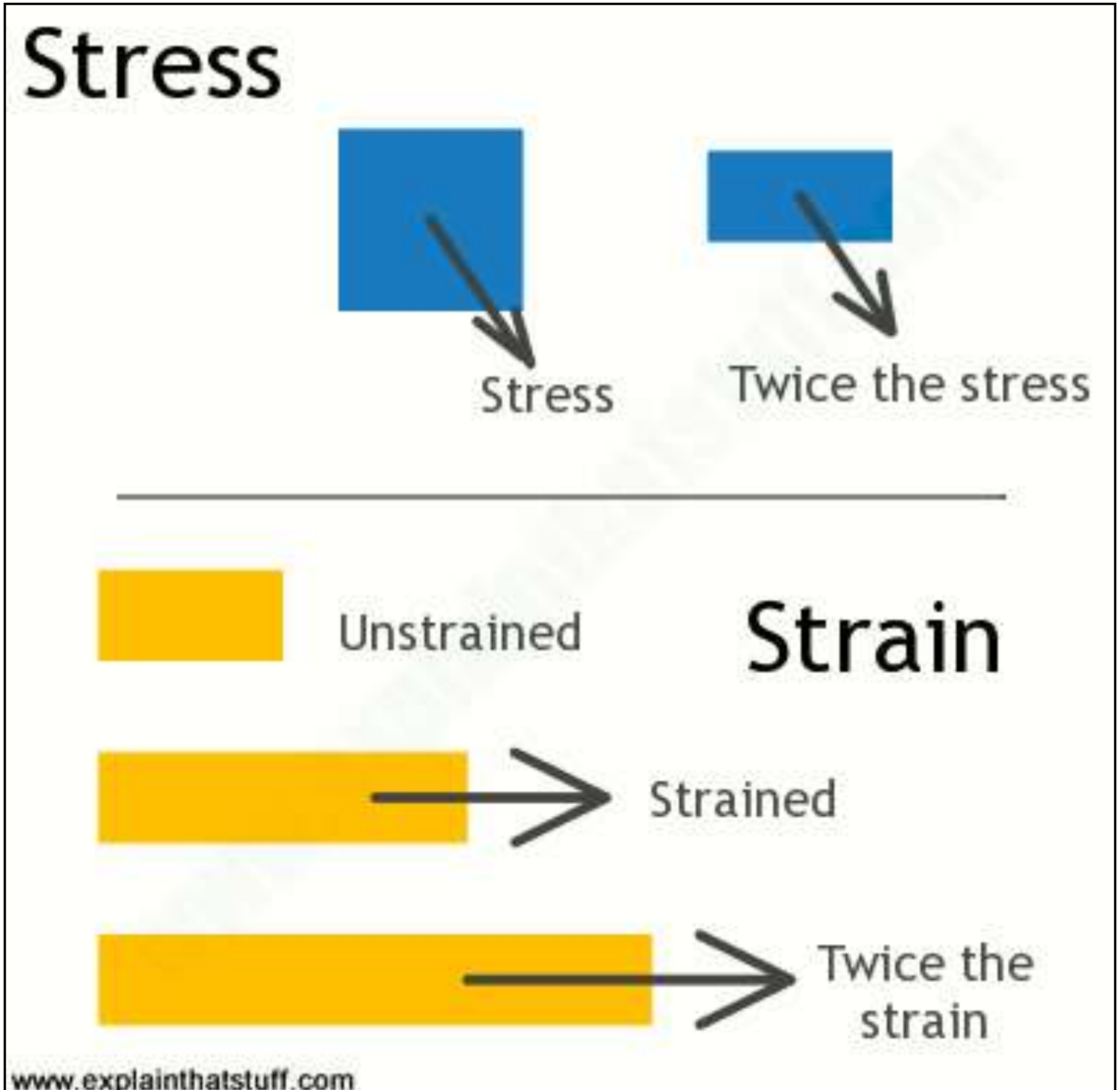
- Is your house in danger of falling down?
- Was that an earthquake?
- Will that airplane actually fly?
- These are just a few of the questions you can study with a handy little device called a strain gauge (sometimes spelled "gage").

- It's a neat way of measuring how much a material changes shape when a force acts on it.
- Strain gauges range from the immensely simple to the fiendishly complex, but all of them are superbly useful to scientists and engineers.

- Are you stressed?
- Can you feel the strain?
- When we talk about "stress" and "strain" in everyday life, we use the two words interchangeably.
- But in science and engineering, these two words have very precise and very different meanings:

- Stress is a measurement of how much *internal pressure* a material is under when a force acts on it.
- The bigger the force or the smaller the area over which it acts, the more likely it is that the material is to going to deform (change shape).
- Stress = Force / Area

- Strain is what happens as a result of stress.
- If a material is stressed by a force, it often changes shape and gets a little bit longer or shorter.
- The strain is defined as the change in length the force produces divided by the material's original length.
- So if you pull a 10cm-long piece of elastic and it stretches by 1cm, the strain is 0.1.


Stress
Stress
Twice the stress
Unstrained
Strain
Strained
Twice the strain
www.explainthatstuff.com

- ***Top: Stress:***
- ***If you apply a pulling force to a bar of a certain cross-sectional area, you create a certain stress.***
- ***If you apply the same force to a bar half the area, you produce twice as much stress.***

- ***Bottom: Strain:***
- ***If you apply no force to a bar, you don't stretch it at all.***
- ***Apply a certain force and you'll extend its length by a certain amount, producing a certain strain.***
- ***If you apply more force so you double the extension, you'll have produced twice as much strain.***

Have you considered the consequences of not measuring strain properly?




Materials Under Stress

- **Different materials behave in very different ways under similar amounts of stress.**
- **If you subject a rubber band to stress, by pulling, it stretches accordingly; release the stress and the band returns to its previous shape.**

- **When materials go back to their original shape and size after stressing forces are removed, they undergone elastic deformation;**
- **including rubber, some plastics, and many metals (which, you might be surprised to hear, are perfectly elastic when very small forces are involved).**

- Eventually, elastic materials reach a point where they can't cope with extra stress and stretch permanently.
- This kind of change is called plastic deformation. (Note that the proper meaning of plastic is something that changes shape relatively easily).

- If you're an engineer, stresses and strains are incredibly important.
- If you're designing anything from a car engine or a bridge to a wind turbine or an airplane wing, you know it's going to be subject to some pretty hefty forces.
- Can the materials you want to use stand up to those forces?

- **Will they deform elastically by tiny amounts and return safely to their original shape and size?**
- **Will they break apart after repeated stresses and strains through a process such as metal fatigue (where repeated deformation causes a metal to weaken and suddenly snap).**

- Do you need to use something stronger to be on the safe side?
- And how exactly can you tell?
- You can do your calculations in the lab and try to figure it out in advance.
- You can even build sophisticated computer models to help you.

- **But the sure-fire way of getting an answer to how materials are coping under pressure is to use strain gauges to measure the way they behave when real-life forces act on them.**

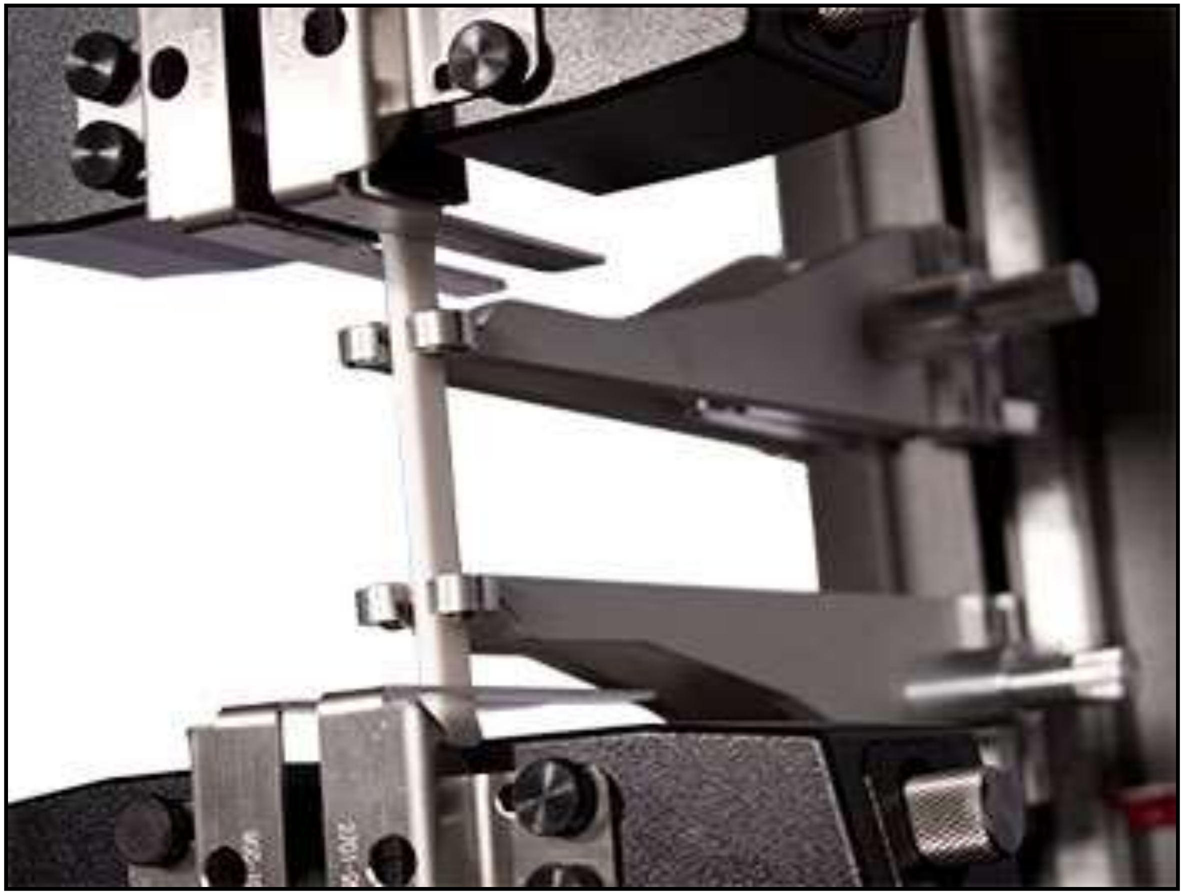
Methods of Strain Measurement

- Contacting Extensometer
- Non-Contacting Extensometer
- Bonded Strain Gauges

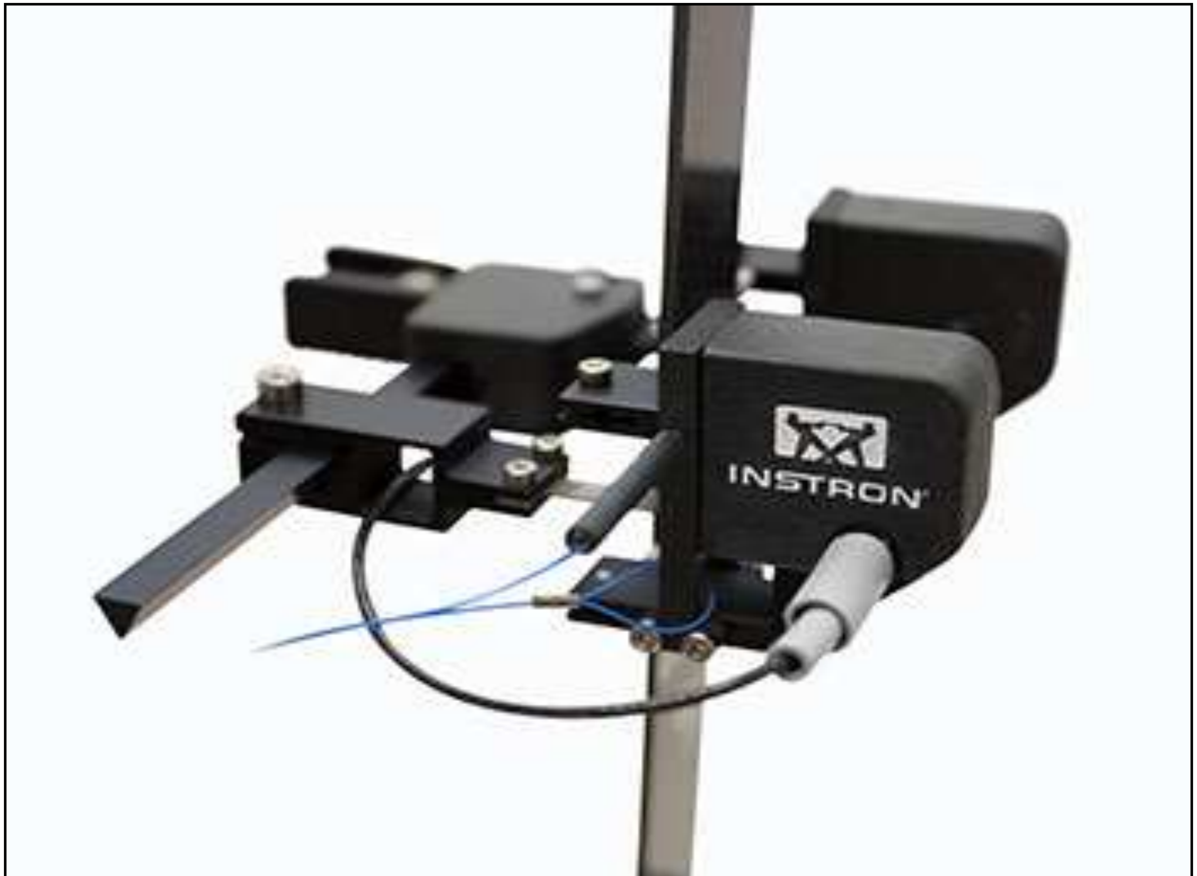
- Strain measurement in materials testing is traditionally carried out using some form of contacting extensometer.
- A typical clip-on extensometer, for example, attaches to the specimen with clips or elastic bands and uses knife-edges to accurately track deformation in a specimen during testing.

- While providing accurate strain measurement in numerous applications, contacting extensometers carry some inherent disadvantages; and therefore,
- it is highly recommended that the user review on "Considerations when Choosing an Extensometer" and carefully consider all aspects of the testing application before selecting the proper device.

Automatic Contacting

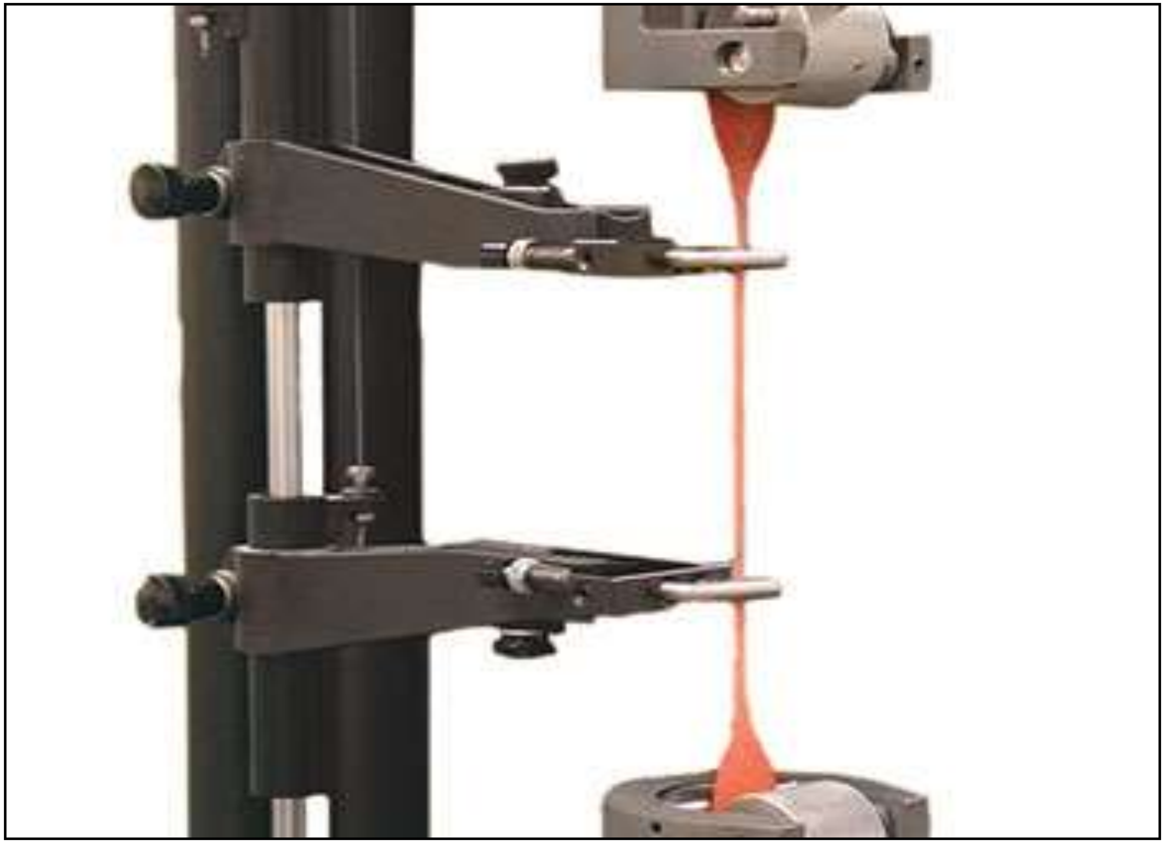


Averaging Axial and Biaxial



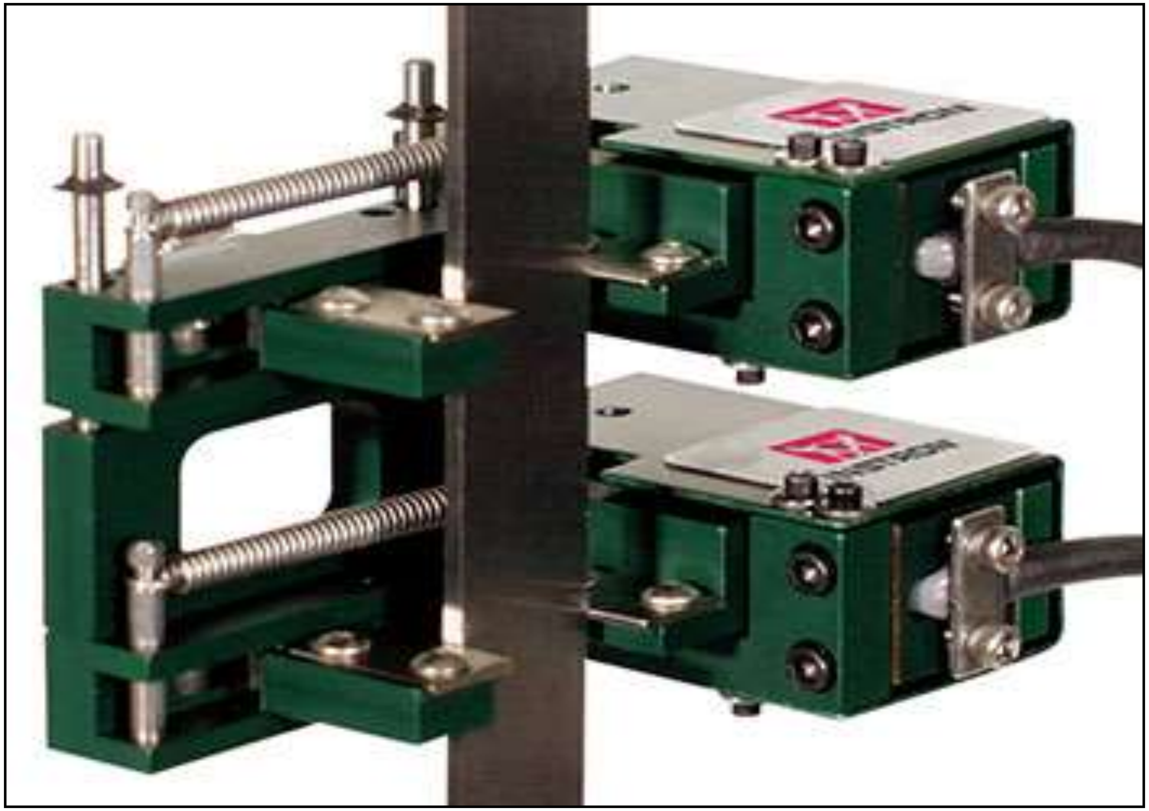
Dynamic clip-on

Long Travel



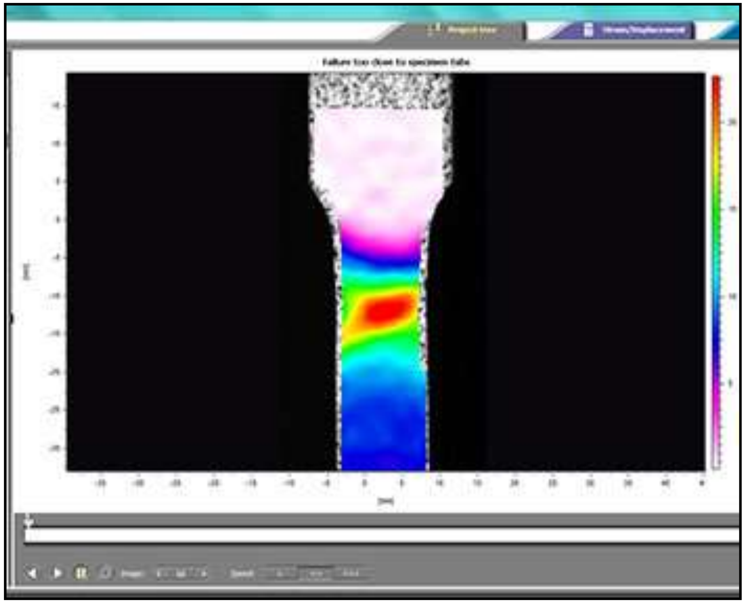
Static clip-on

Transverse



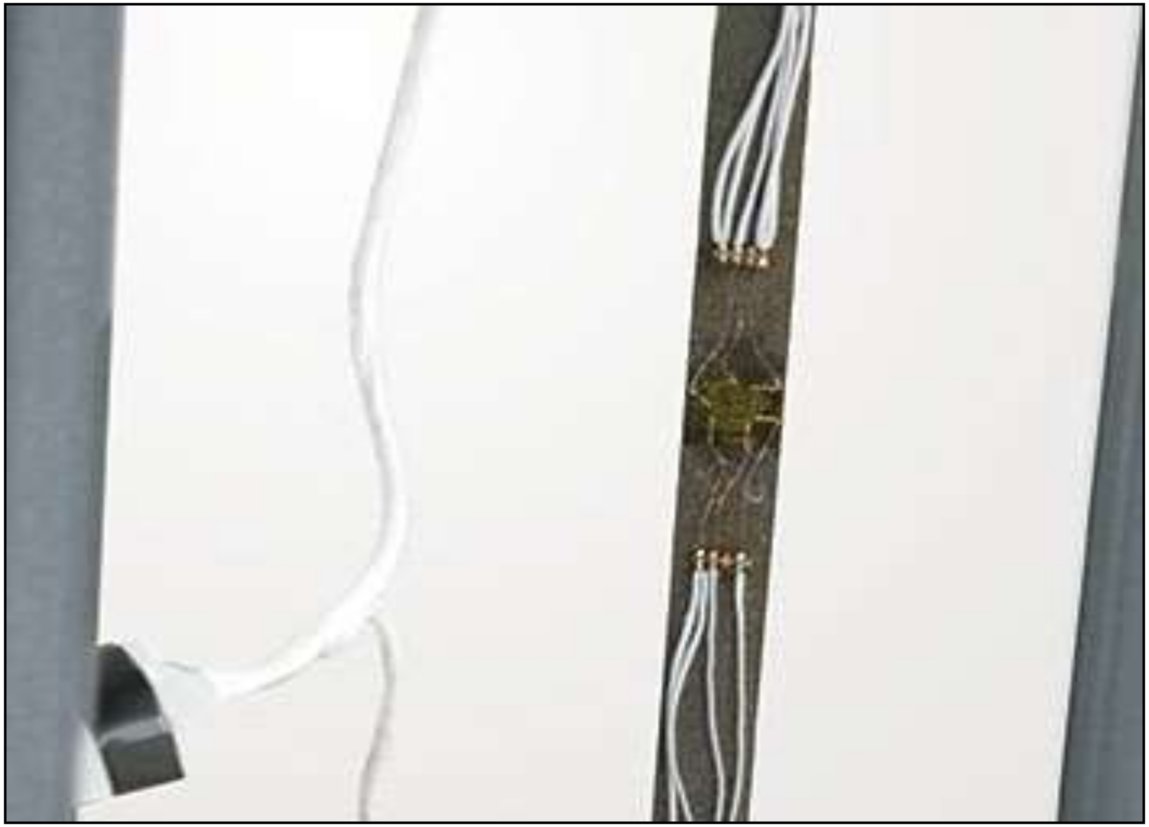
- Several methods of non-contacting strain measurement exist, ranging from the most basic method of measuring displacement of the crosshead to full-field strain mapping using Digital Image Correlation (DIC).

Non-Contacting Extensometer



BONDED STRAIN GAUGES

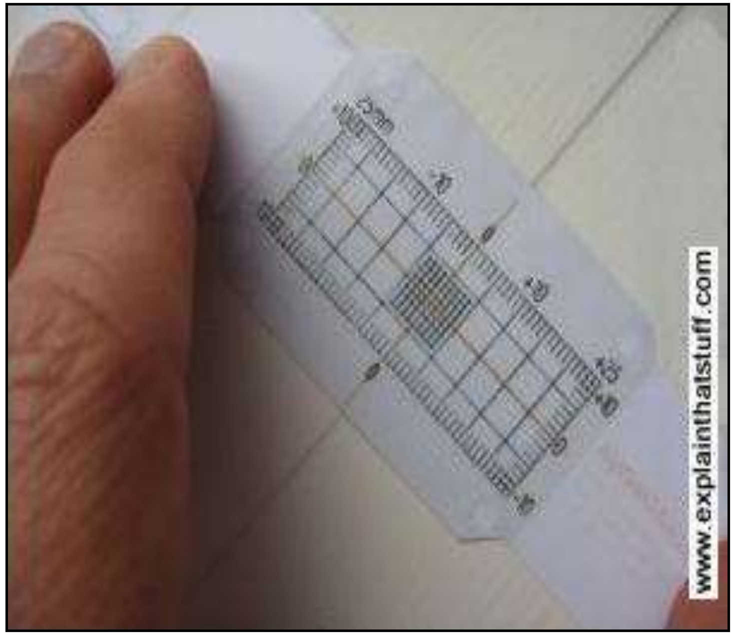
- Most clip-on contacting extensometers use strain gauges to measure strain on the specimen, but to achieve higher accuracy the strain gauge itself can be applied directly to the surface.



BONDED STRAIN GAUGES

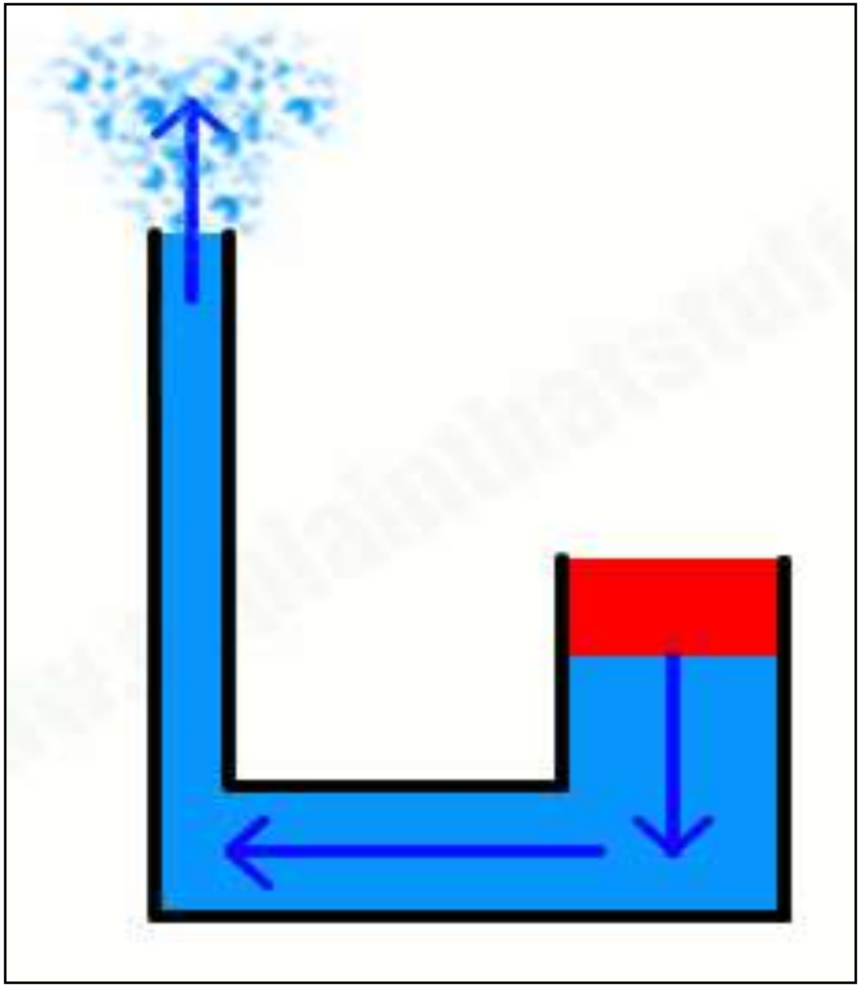
Mechanical

- Suppose you have a crack forming in a wall of your home because of subsidence and you want to know if it's getting any worse.
- Call in the building inspectors and they'll probably glue a piece of tough, flexi glass plastic, ruled with lines and a scale, directly over the crack.



- *A simple mechanical crack monitor. You watch the red crosshairs move on the scale as the crack widens. Detectors like this are made by companies.*

Hydraulic



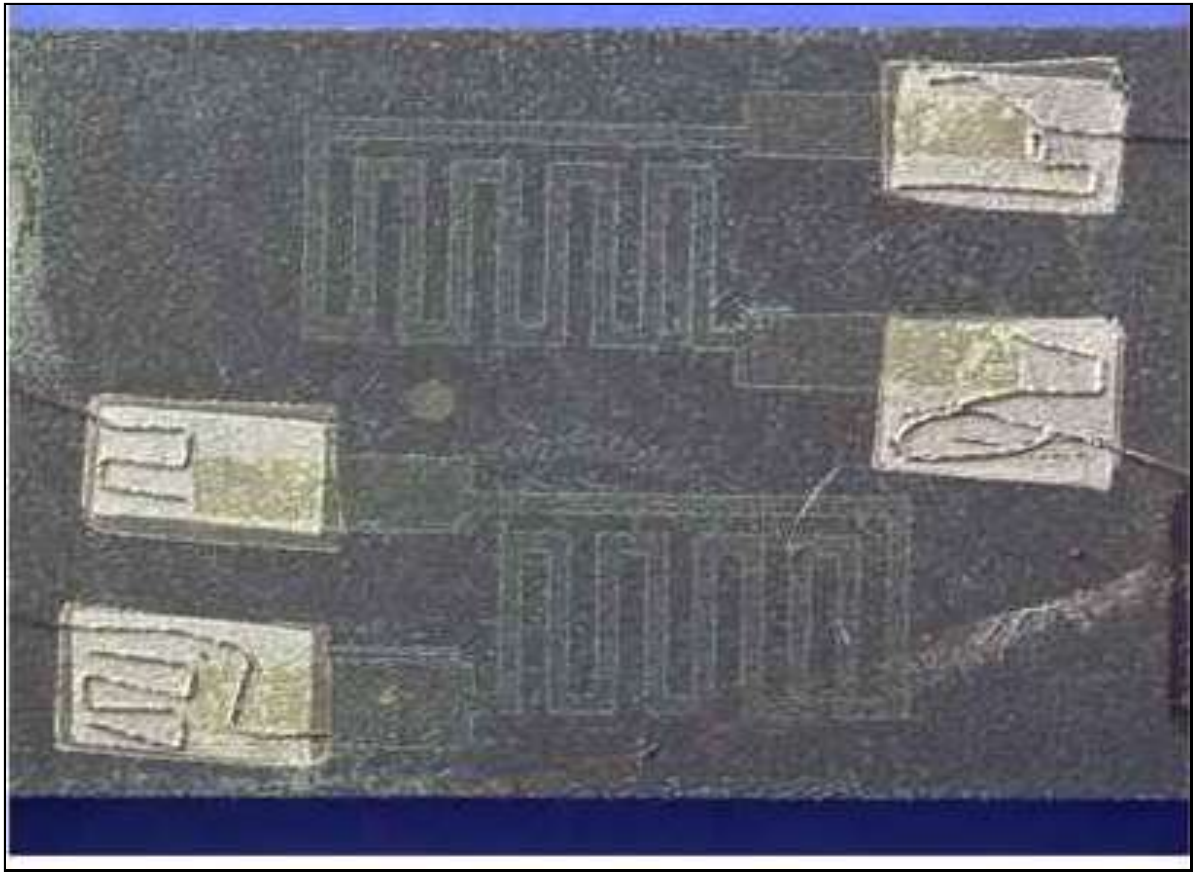
The basic principle of a hydraulic strain gauge

- ***we can use hydraulics to amplify the movement caused by a small strain, making it easier to measure more precisely.***
- ***If a certain force presses down on the large piston (red) on the right, it moves it down a certain distance.***

- ***Because liquids are largely incompressible and the amount of fluid moving must remain constant, the smaller piston in the tube on the left must move up a greater distance.***
- ***This is the basic theory behind hydraulics, known as Pascal's principle.***

- **Hydraulic detectors offer a solution and work much like simple syringes.**
- **Syringes are essentially hydraulic pistons where a small movement of fluid in a large piston (the part you press with your finger) produces a much larger movement of fluid in a small piston attached to it (the needle where the fluid comes out).**

Electrical Strain Gauges



- If you're designing like an airplane wing, typically you need to make far more sophisticated measurements than a simple mechanical strain gauge.
- You might want to measure the strain during takeoff, for example, when the engines are producing maximum thrust.

- You can't go sticking little plastic strain gauges onto the wing and walk out to measure them during a flight!
- But you can use electrical strain gauges to do much the same thing from a flight recorder in the cockpit.

- **The most common electrical strain gauges are thin, rectangular-shaped strips of foil with maze-like wiring patterns on them leading to a couple of electrical cables.**
- **You stick the foil onto the material you want to measure and wire the cables up to your computer or monitoring circuit.**

- When the material you're studying is strained, the foil strip is very slightly bent out of shape and the maze-like wires are either pulled apart or pushed together.
- Changing the width of a metal wire changes its electrical resistance, because it's harder for electrons to carry electric currents down narrower wires.

- So all you have to do is measure the resistance and, with a bit of calculation, you can calculate the strain.
- If the forces involved are small, the deformation is elastic and the strain gauge eventually returns to its original shape—
- so you can keep making measurements over a period of time, such as during the test flight of a prototype plane.

- **Resistance-type strain gauges were invented in 1938 by MIT professor Arthur Ruge (1905–2000) to help with earthquake detection.**

Piezoelectric

- ***A piezoelectric quartz oscillator from a watch changes shape thousands of times a second when electricity flows through it.***
- ***In a piezoelectric strain gauge, the crystal works the opposite way—generating electricity as its shape changes.***
- ***The bigger the strain, the bigger the current the crystal generates so measuring the current is a way of accurately measuring the strain.***

- Some types of materials, including quartz crystals and various types of ceramics, are effectively "natural" strain gauges.
- If you push and pull them, they generate tiny electrical voltages between their opposite faces.
- This phenomenon is called piezoelectricity

- **Measure the voltage from a piezoelectric sensor and you can calculate the strain very simply.**
- **Piezoelectric strain gauges are among the most sensitive and reliable and can withstand years of repeated use.**
- **sometimes called piezoelectric transducers, because they convert mechanical energy into electrical energy.**

M.E I SEMESTER (STRUCTURAL ENGINEERING) SEE 104:EXPERIMENTAL STRESS ANALYSIS AND INSTRUMENTATION

Reasons for Experimental Stress Analysis

- Material characterization
- Failure analysis
- Residual or assembly stress measurement
- Acceptance testing of parts prior to delivery or use

Some Techniques

- ***Electrical Resistance Strain Gauges***
- **Photo-elasticity**
- **Non-contact holographic interferometry**

Basic Concepts of Stress and Strain Analysis

- Concept of stress
- Stress at a point
- Normal and Shear Stress
- Equality of cross shears
- Principle stresses and principle strains

- State of pure shear
- State of pure bending (2 point load)
- Differential equations of equilibrium
- AIRY'S Stress function
- Compact-ability conditions: Generalized Hook's law

Stress vs Strain

- **Strain (ε)** is a measure of displacement usually in terms of micro-strain such as micro-inches of elongation for each inch of specimen length.
- **Stress (σ)** is a measure of loading in terms of load per unit cross sectional area.

- Stress and strain are related by a material property known as the **Young's modulus** (or modulus of elasticity) E.

$$E = \sigma / \varepsilon$$

- **Strain Defined:**
- **Strain is defined as relative elongation in a particular direction**
- **ε_a= dL/L (axial strain)**
- **ε_t= dD/D (transverse strain)**
- **$\mu = \varepsilon_t / \varepsilon_a$ (Poisson's ratio)**

- **Strain Gauges:**
- **The electrical resistance of a conductor changes when it is subjected to a mechanical deformation**
- **Resistance = f(A…) = R**

- Electrical Resistance (R) is a function of…
- ρ the resistivity of the material (Ohms*m)
- L the length of the conductor (m)
- A the cross-sectional area of the conductor (m^2)
  - $R = \rho * L/A$

- Note R increases with
- – Increased material resistivity
- – Increased length of conductor (wire)
- – Decreased cross-sectional area (or diameter)
- – Increased temperatures

Measurements

- Principles of measurements
- Accuracy
- Sensitivity
- Range of measurements

- **Stress analysis:** It is an Engineering discipline that determines the stress in materials & structures subjected to static or dynamic forces (or) loads.
- **Aim of the analysis:** To determine whether the [element or collection of elements] “STRUCTURE” can safely with stand the specified forces.

- **Normally the safety load can be measured using F.O.S [factor of safety]**

$$= \frac{\text{ultimate stress}}{\text{maximum allow the stress}}$$

- **This F.O.S. given to design engineers for the purpose of design. From the F.O.S the design Analyst calculate design factor.**

- **“Design factor “**

$$= \frac{\text{ultimate tensile stress}}{\text{Maximum calculator tensile stress}}$$

- **Types of load acting on a structure:**
- *Tension
- *Compression
- *Shear
- *Torsion
- *Bending

Software Used By Design Engg

- * Pro - Mechanica
- * Analysis
- * Misc software
- * Nastron
- * Rohrz [analysis software]
- * Caesar II

- **Define Measurement**
- **The process of obtaining the magnitude of a quantity such as length or mass relative to a unit of measurement such as meters or kilogram.**
- *** The act of measuring or the process of being measured [used]**
- *** The system of measuring**

- **TYPES OF MEASUREMENT:**
- **Generally two measurements**
- **VECTOR'S : have an magnitude [an amount] & a direction**
- **SCALAR'S : have an magnitude but have no direction.**
- **All measuring instruments have calibrations. These are markings or division in measuring tool.**

- On the basis of S.I units the measure divided & classified into following:

  - Linear [length or distance]
  - Mass [weight]
  - Volume
  - Temperature

- Linear measurements are made using a Metric stick or Metric Ruler. Measured in meter, centimeter, millimeter.
- Mass measurements are made using a balance: Triple beam balance–Dial gram balance–Electric/ digital balance-Analytical balance. Measured in gram, kilograms, centigrams, milligrams.

- **Volume:** The volume of any solid, liquid, gas, plasma or vacuum is how much 3-D space it occupies. **Measured in cubic meters, cubic centimeter liters, milliliters.**
- **Temperature:** Temperature measurement using modern scientific thermometers & temperature scales. **Measured in Fahrenheit, Kelvin, Celsius.**

- **Principles of Measurement:**
- **The techniques of measurement are of immense importance in most facets of scientific research & human civilization.**
- **Computation with decimals frequently involves the addition or subtraction of numbers do not have the same number of decimal places.**

- **Estimation:**
- **Estimation is the calculated approximation of a result which is usable even if input data may be incomplete or uncertain.**
- **It can be computed precisely.**

- **Precision:**
- **The Measurement of a precision depends upon how precisely the instrument is marked.**
- **It is important to realize that precision refers to the size of the smallest division on the scale.**
- **The precision of measurement system also called reproducibility or repeatability.**

- **Reproducibility:** It is one of the main principles of the scientific method & refers to the ability of a test or experiment to be accurately reproduced.
- **Repeatability:** It is the variation in measurement taken by a single, person or instrument on the same item & under the same conditions.

- **Accuracy:**
- **The accuracy of measurement depends upon the relative size of the probable error.**
- **The Accuracy of a measurement system is the degree of closeness of measurements of a quantity to its actual [true] value.**
- **The measurement system is valid if it is both accurate & precise.**

$$\text{ACCURACY} = \frac{\text{No of true positives} + \text{no of true negatives}}{\text{no of true positives \& false positives} + \text{false negatives} + \text{true negatives}}$$

$$\text{Precision} = \frac{\text{No of true positives}}{\text{No of true positives} + \text{false positives}}$$

$$\text{sensitivity} = \frac{\text{No of true positives}}{\text{No of true positives} + \text{no of false negatives}}$$

- **Specificity:**

$$\frac{\text{No of true negatives}}{\text{No of true negatives} + \text{no of false positives}}$$

- Example:
- True positives (TP) – sick people correctly diagnose as sick
- False positives (FP) _ Healthy as sick
- True Negatives (TN) _ Healthy correctly indentified as healthy
- False negatives (FN)_ Sick people incorrectly identified as healthy
- False positives & False negatives also called as Type –I & Type II error
- TP $\rightarrow$ condition present + positive result
- FP $\rightarrow$ condition absent + positive result
- FN $\rightarrow$ condition present + Negative result
- TN $\rightarrow$ condition absent + Negative result

- **Factors to be governed while selecting a strain gauge:**
- **Gauge length**
- **Sensitivity**
- **Range**
- **Accuracy or Repeatability**
- **Readability**
- **Require skilled operator**
- **Cost**

- **Gauge Length**
- **The distance between the two knife edges in contact with the specimen and by the width of the movable knife edges.**
- **The gauge length becomes more important when measuring long linear strain should be as far as possible in that case.**

- **Sensitivity**
- **It is the smallest value of the strain which can be read on the scale associated with the strain gauges (minimum).**

- **Range (Maximum)**
- **This represents the maximum strain which can be recorded without resetting or replacing in the strain gauge.**
- **The range and sensitive gauges response to small strain with applicable deflections.**
- **The range is mutually limited to the full scale range to the deflector.**

- **Accuracy or Repeatability**
- **Sensitivity does not ensure accuracy usually the very sensitive instrument are quit prone to errors**
- **unless they are employed with a design could use components that are light in weight are possible.**
- **So that the forces require to support as far as possible.**

Characteristic of an ideal strain gauge:

- The gauge should be of extremely small size of gauge (length and width) as to estimate strain at a point, the gauge should be as significant mass to permit the recording of dynamic gauge.
- The gauge should be easy to attach the member being the analyzed on the easy to handle.

- The strain sensitivity and accuracy of the gauge should be sufficiently high.
- The gauge should not be affected by temperature , vibration, heat, etc.
- The gauge should be capable of indicating work starting and dynamic strength.
- It should be possible to read the gauge either in location or remotely.

- The gauge should exhibit linear response to strain.
- The gauge and associated equipment should be available at a reasonable cost.
- The gauge should be suitable for use as a sensing element or other transducers system.

- The following factors should be considered while selecting a gauge:
- Gauge Material
- Grid Geometry
- Grid Configuration
- Gauge factors
- Gauge size
- Gauge Resistance

- Gauge Resistance
- Gross sensitivity
- Lead out- method
- Gauge material form
- Aggressive used
- Type of strain to be measured
- Operating temperature and Environmental condition

- Different type of strain gauge
- Mechanical strain gauge
  - Lever type (Simple and component)
  - Rack and pinion type
  - Combination of rack and pinion type
- Optical strain gauge

- Electrical Strain gauge
    - ❖ Inductance type
    - ❖ Capacitance type
    - ❖ Resistance type
    - ❖ Piezo electric type
- Pneumatic Strain gauge
- Acoustical strain gauge

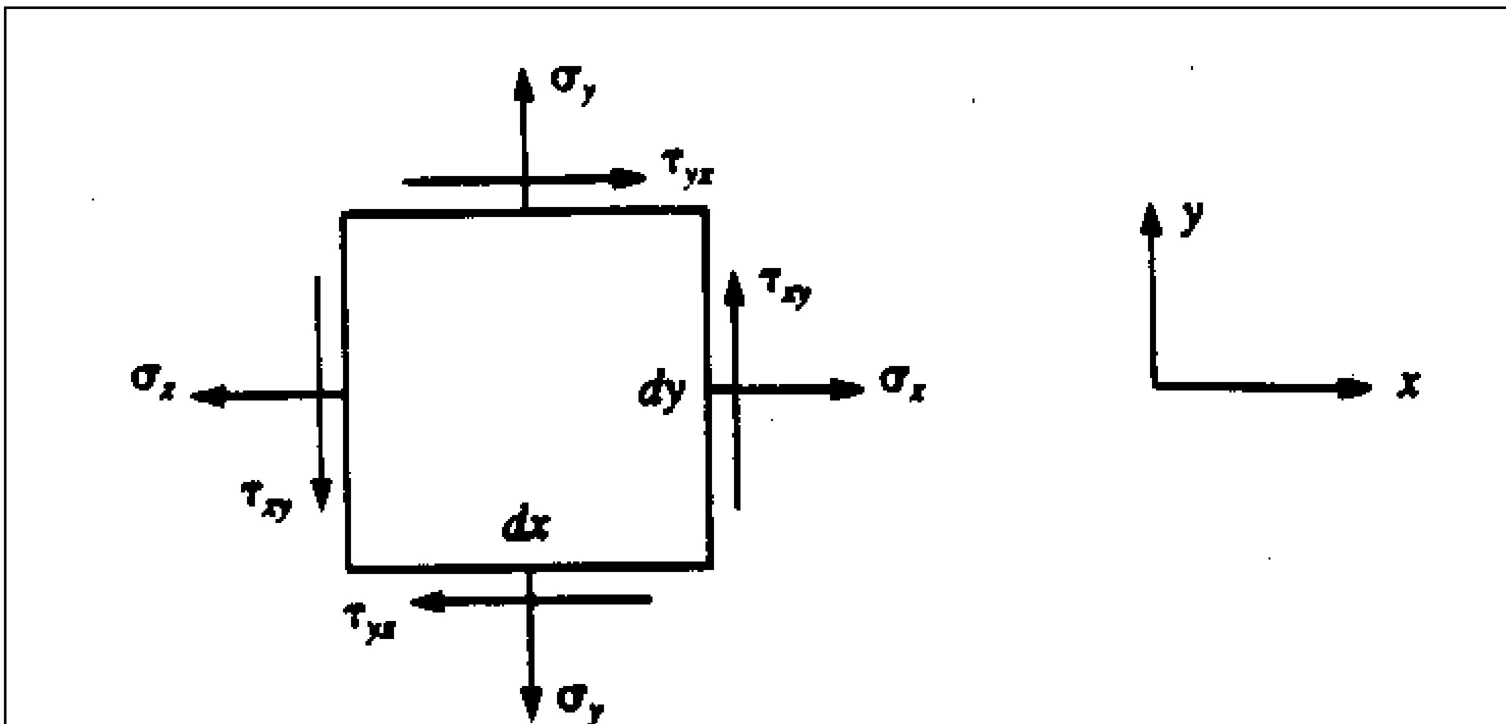
MODULE
for
Plane Stress
and
Plane Strain
Analysis

TWO-DIMENSIONAL ELASTICITY

- **Many problems in elasticity may be treated satisfactorily by a two-dimensional, or plane theory of elasticity.**
- **There are two types of problems involved in this plane analysis, plane stress and plane strain.**

- These two types will be defined by setting down certain restrictions and assumptions on the stress and displacement fields.
- They will also be introduced descriptively in terms of their physical prototypes.

The two dimensional state of stress is illustrated below where σ_x and σ_y are normal stresses and τ_{xy} and τ_{yx} are the shear stresses.



As seen three independent stresses, σ_x, σ_y and τ_{xy} exist which can be written as the following stress vector:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}$$

- **From these stresses the max and min normal stresses, the principal stresses, in the two-dimensional plane are:**

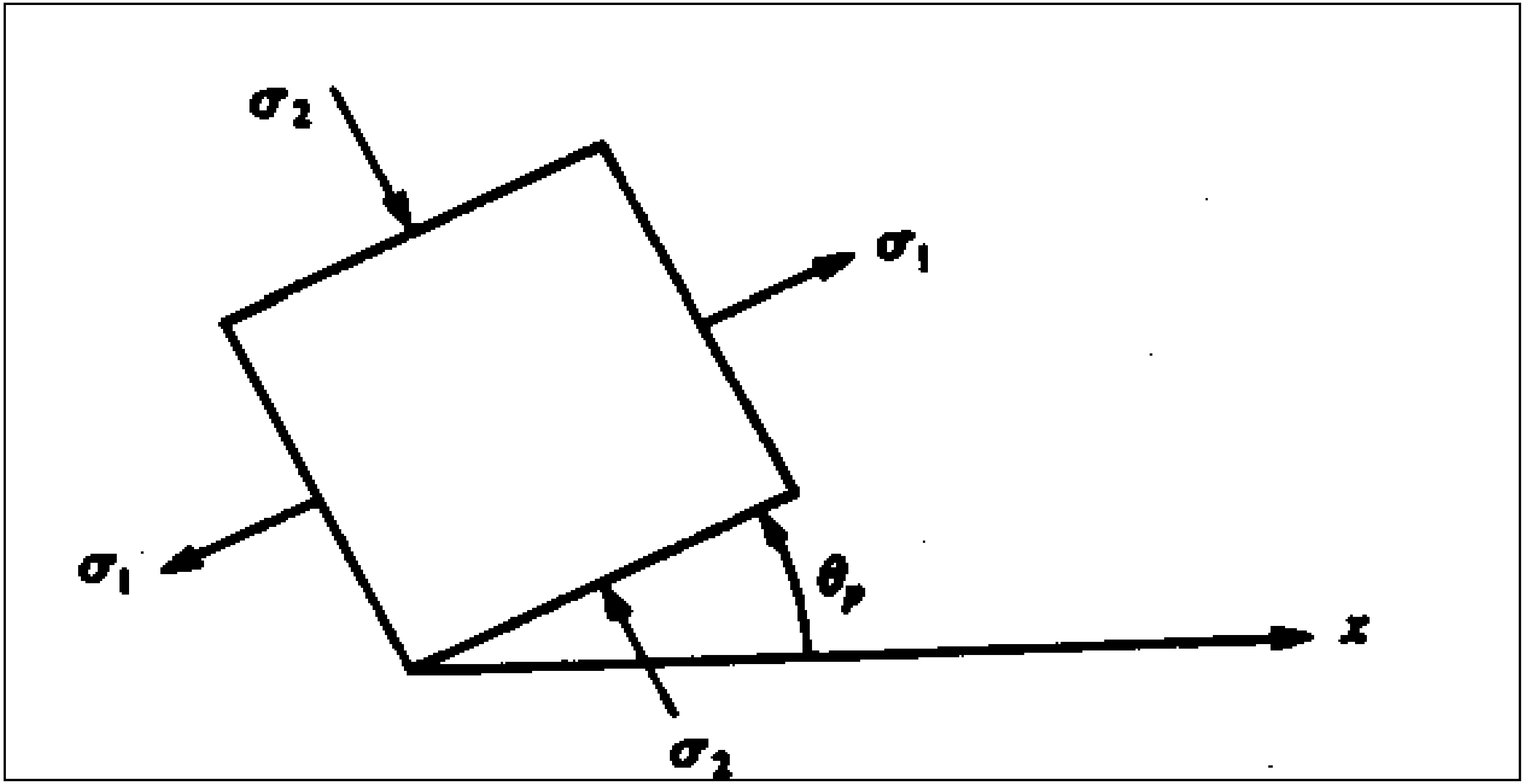
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{max}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sigma_{min}$$

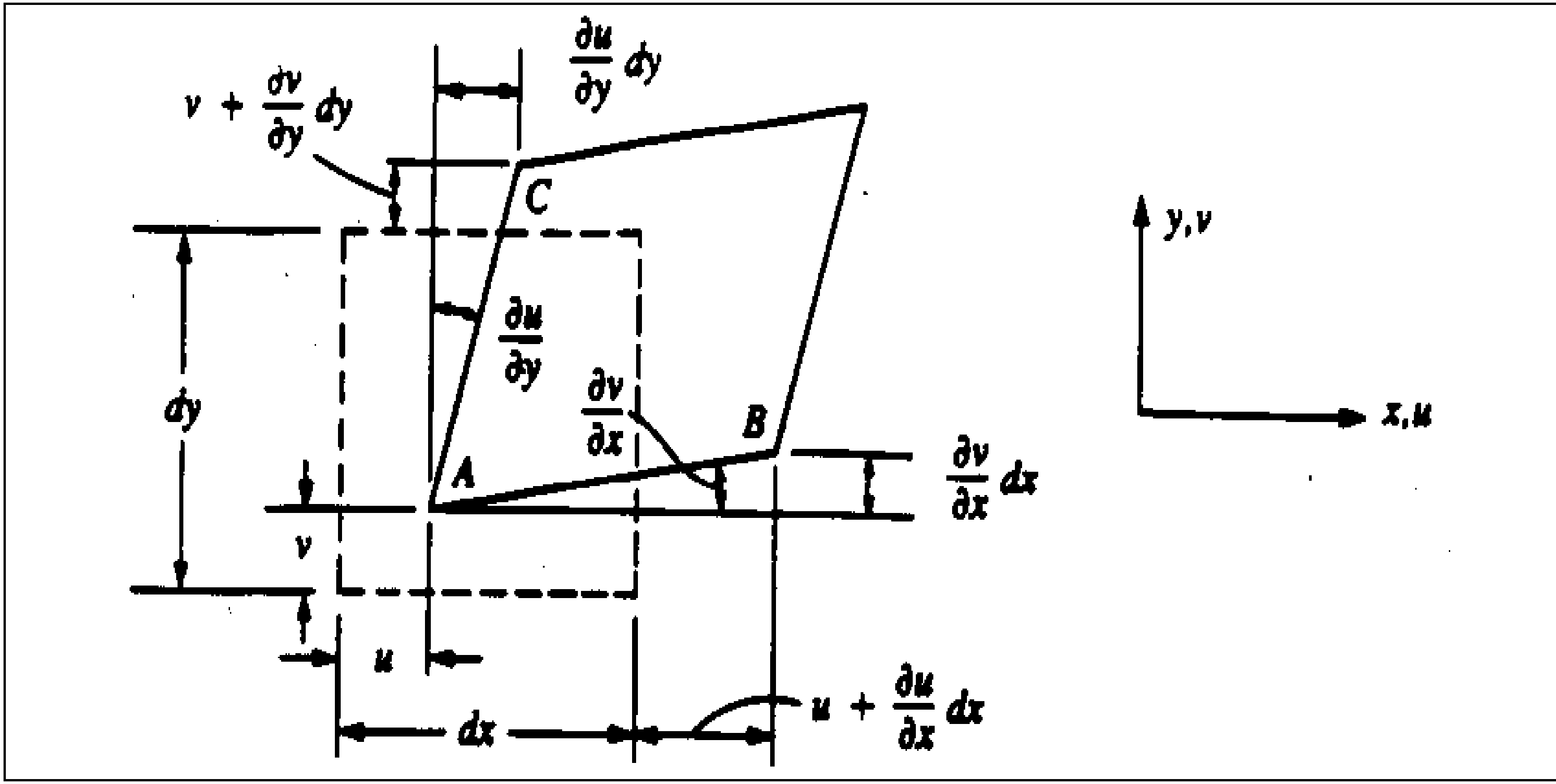
and the principle angle is

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The principal stresses and their directions are shown below:



The general two-dimensional state of strain at some point in a structure is represented by
the shown infinitesimal element, dx dy,
where u and v are the x and y-displacements at point A, respectively, and
lines AC and AB have been extended and displaced.



$\frac{\partial u}{\partial y}dy$
$v + \frac{\partial v}{\partial y}dy$
C
$\frac{\partial u}{\partial y}$
dy
$\frac{\partial v}{\partial x}$
B
A
$\frac{\partial v}{\partial x}dx$
v
u
dx
$u + \frac{\partial u}{\partial x}dx$
y,v
x,u

- **The normal (ex-tensional or longitudinal) strains are defined as**

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y}$$

and the shear strain is

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

These can be written as the strain vector

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

The basic partial differential equations for plane elasticity including body and inertia forces are:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + X = \rho \frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = \rho \frac{\partial^2 v}{\partial t^2}$$

- **where X and Y denote the body forces per unit volume in the x and y-directions, respectively, and**
- **ρ is the density of the material.**

Plane Stress

Plane stress is defined to be a state of stress in which the normal stress, σ_z, and the shear stresses, τ_{xz} and τ_{yz} directed perpendicular to the x-y plane are **assumed to be zero.**

- The geometry of the body is essentially that of a plate with one dimension much smaller than the others.
- The loads are applied uniformly over the thickness of the plate and act in the plane of the plate as shown.

- The plane stress condition is the simplest form of behavior for continuum structures and represents situations frequently encountered in practice.

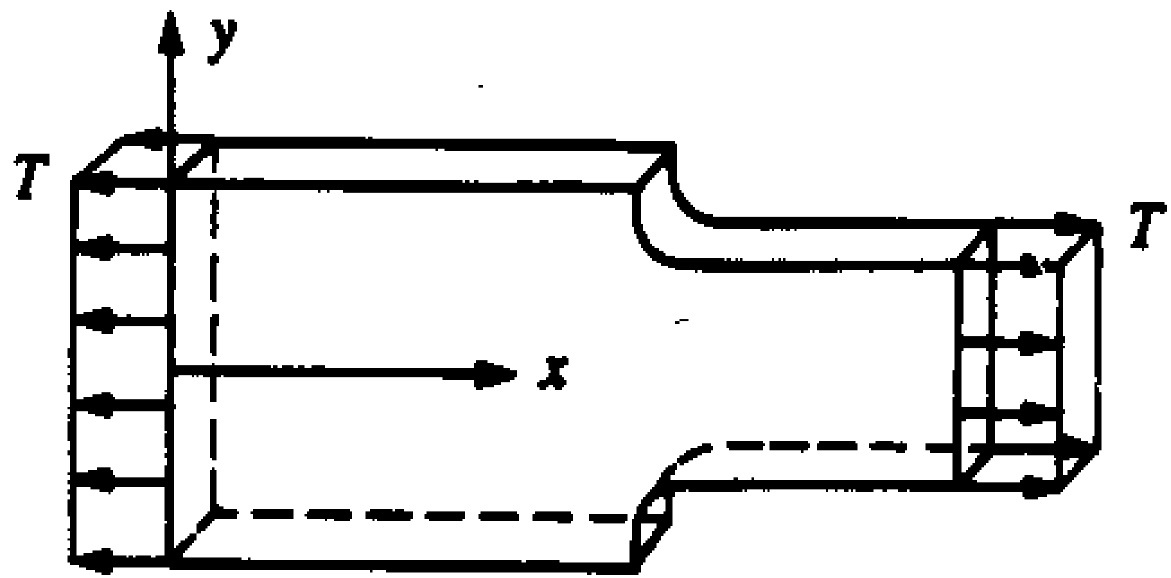


plate with fillet

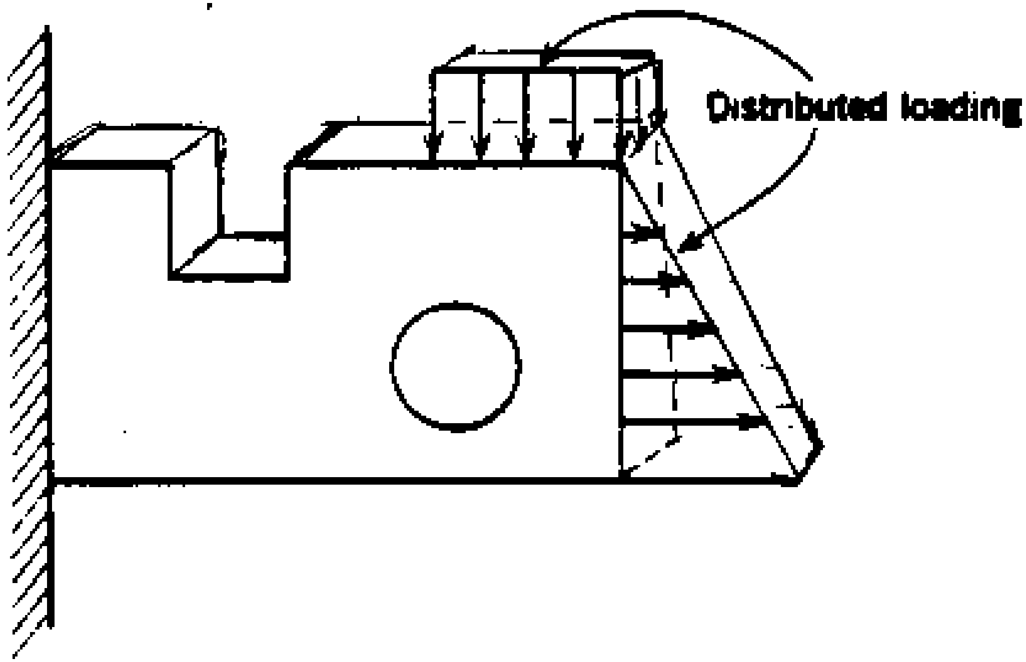
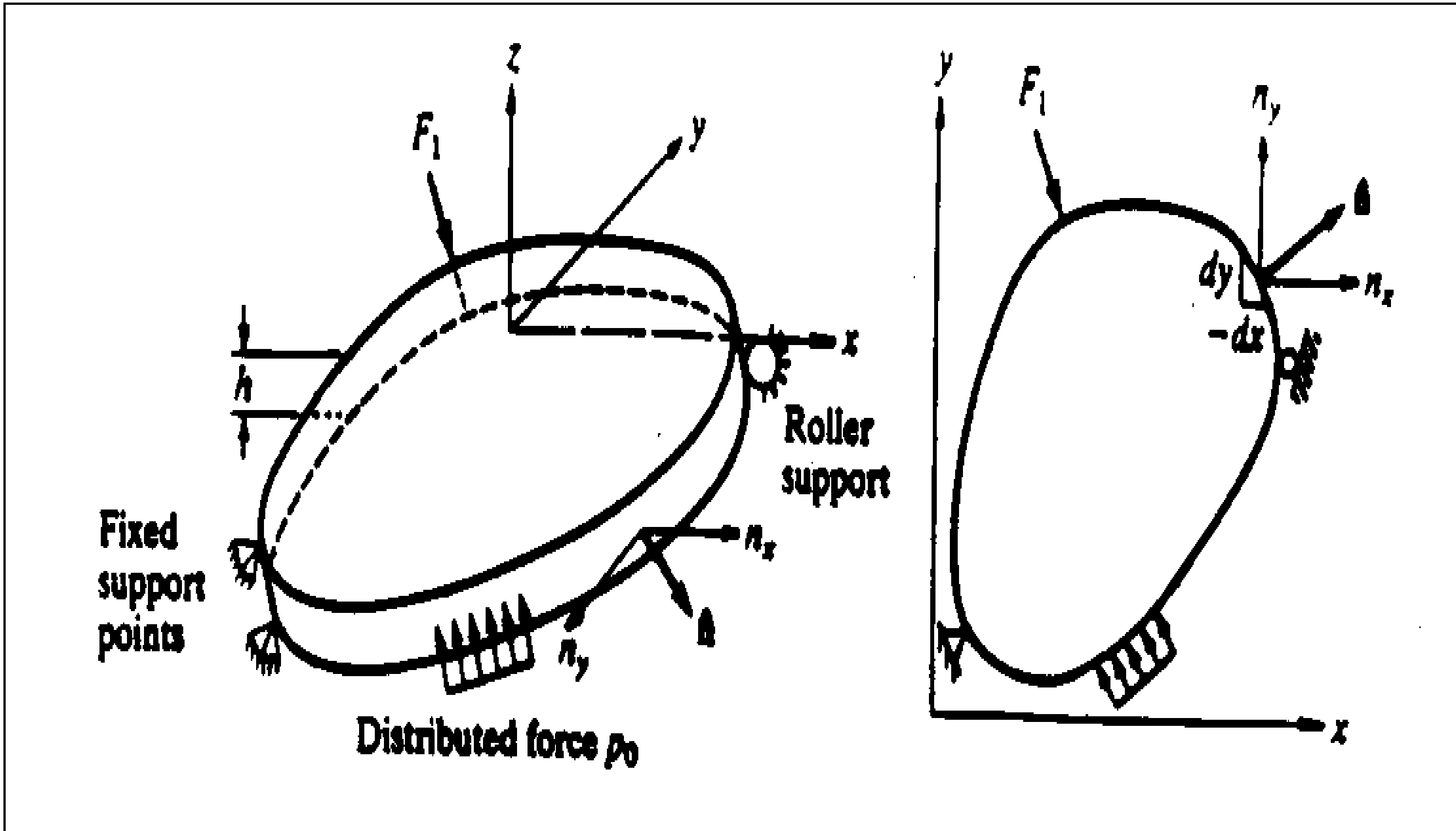


plate with hole

Typical loading and boundary conditions for plane stress problems in two-dimensional elasticity.

a) **Loadings may be point forces or distributed forces applied over the thickness of the plate.**

b) **Supports may be fixed points or fixed edges or roller supports.**



z
y
x
F_1
h
Roller support
Fixed support points
n_x
n_y
n
Distributed force p_0
y
F_1
n_y
n
n_x
dy
−dx
x

Support conditions:

$u = v = 0$ $\quad$ $u = 0, v \neq 0$

For isotropic materials and assuming

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0$$

and

$$\gamma_{xz} = \gamma_{yz} = 0,$$

yields

$$\{\sigma\} = [D]\ \{\varepsilon\}$$

where

$$[D] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}$$

in which [D] is the stress/strain matrix (or constitutive matrix), E is the modulus of elasticity and ν is Poisson's ratio.

The strains in plane stress are

$$\{\varepsilon\} = [C]\{\sigma\}$$

or
$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & 2(1+\nu) \end{bmatrix}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}$$

where $[C]^{-1} = [D]$. Also

$$\varepsilon_y = \frac{1}{E}(-\nu)(\sigma_x + \sigma_y)$$

- **The basic partial differential equations for plane stress including body and inertia forces are,**

$$G\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+G\frac{1+v}{1-v}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+X=\rho\frac{\partial^2 u}{\partial t^2},$$

$$G\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+G\frac{1+v}{1-v}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+Y=\rho\frac{\partial^2 v}{\partial t^2}.$$

where the shear modulus G is defined as

$$G=\frac{E}{2(1+v)}$$

Plane Strain

Plane strain is defined to be a state of strain in which the strain normal to the x-y plane, ε_z , and the shear strain γ_{xz} and γ_{yz} are assumed to be zero.

In plane strain, one deals with a situation in which the dimension of the structure in one direction, say the z-coordinate direction,

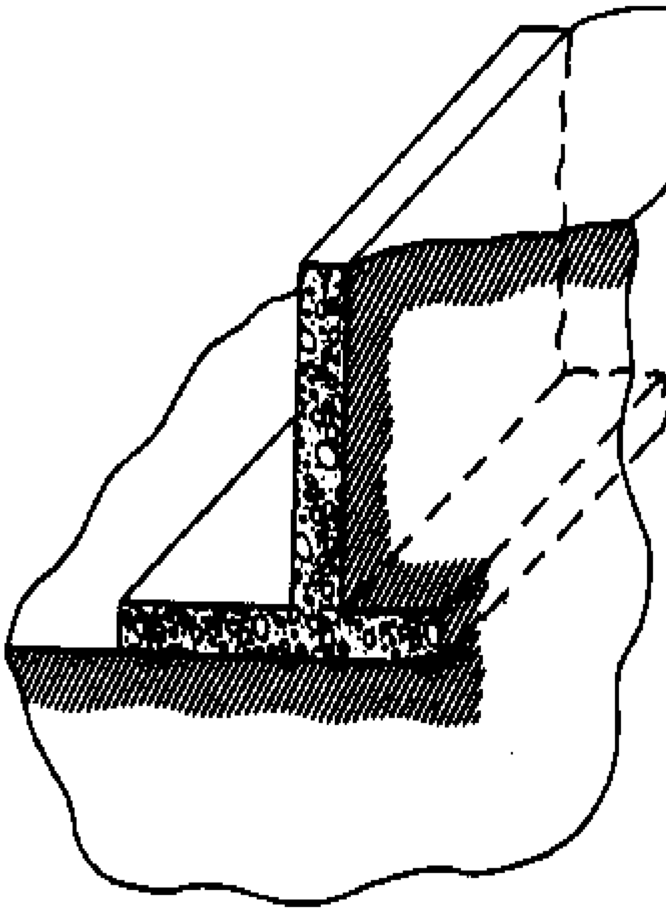
- **is very large in comparison with the dimensions of the structure in the other two directions (x-and y- coordinate axes),**
- **the geometry of the body is essentially that of a prismatic cylinder with one dimension much larger than the others.**

The applied forces act in the x-y plane and do not vary in the z direction,

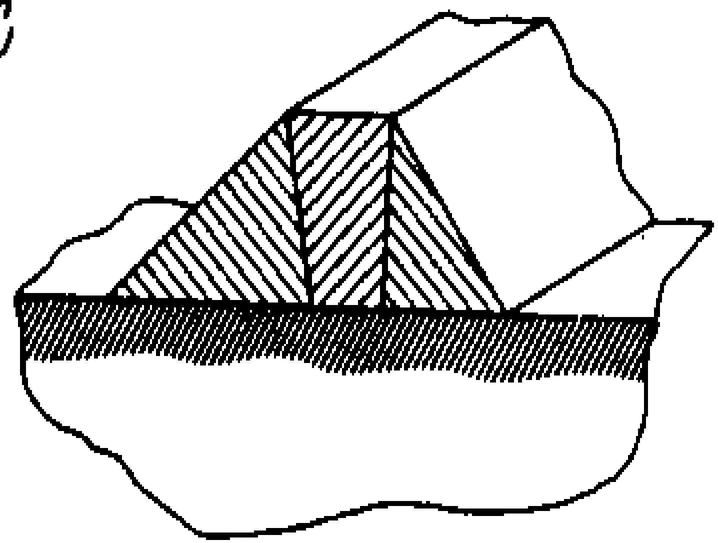
i.e. the loads are uniformly distributed with respect to the large dimension and act perpendicular to it.

Some important practical applications of this representation occur in the analysis of dams, tunnels, and other geotechnical works.

Also such small-scale problems as bars and rollers compressed by forces normal to their cross section are amenable to analysis in this way.

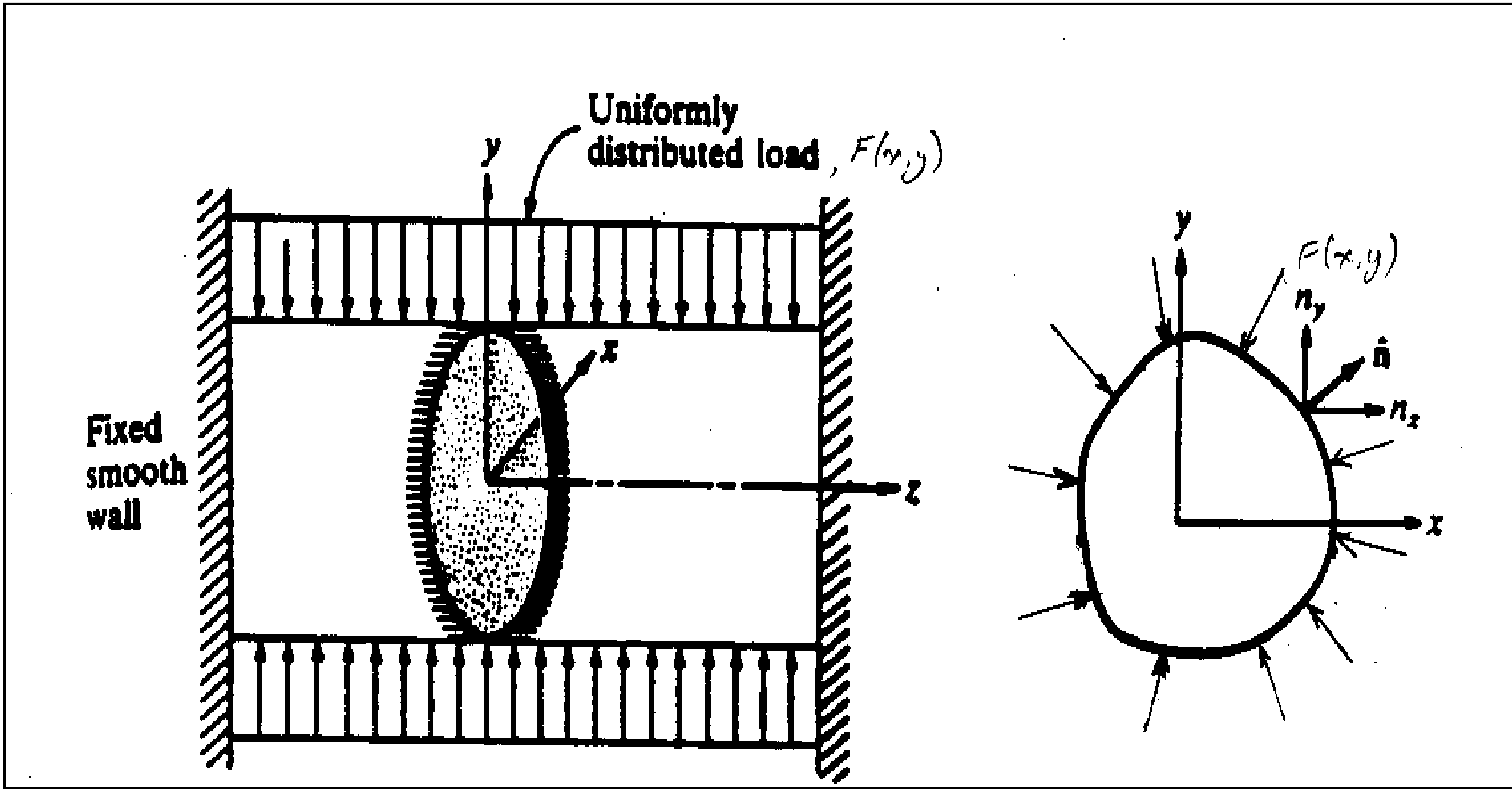


Retaining wall



Earth dam

Typical loading and boundary conditions for plane strain problems in two-dimensional elasticity.


Uniformly
distributed load , F(x,y)
Fixed
smooth
wall
y
x
z
y
F(x,y)
n_y
$\hat{n}$
n_x
x

For isotropic materials and assuming

$$\varepsilon_z = \gamma_{xz} = \gamma_{yz} = 0$$

and

$$\tau_{xz} = \tau_{yz} = 0$$

yields

$$\{\sigma\} = [D]\,\{\varepsilon\}$$

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

with

$$\sigma_z = \frac{E}{1+\nu}\left[\frac{\nu}{1-2\nu}\left(\varepsilon_x + \varepsilon_y\right)\right]$$

The basic partial differential equations for plane strain including body and inertia forces are

$$G\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{1}{1-2\nu}G\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+X=\rho\frac{\partial^2 u}{\partial t^2},$$

$$G\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{1}{1-2\nu}G\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+Y=\rho\frac{\partial^2 v}{\partial t^2}.$$

Where

$$G=\frac{E}{2(1+\nu)}$$

- **Behavioral Assumptions**
- **A plate loaded in its mid-plane is said to be in a state of *plane stress, or a membrane state, if the following assumptions hold:***
- **1. All loads applied to the plate act in the mid-plane direction, and are symmetric with respect to the mid-plane.**

- **2. All support conditions are symmetric about the mid-plane.**
- **3. In-plane displacements, strains and stresses can be taken to be uniform through the thickness.**
- **4. The normal and shear stress components in the *z direction are zero or negligible.***

Two Dimensional Elasticity

- Two and three dimensional elasticity problems are governed by a system of coupled second order differential equations.
- The main variables are the displacements along the coordinate directions.
- Once the displacements are known, stresses and strains can easily be calculated.

Governing Differential Equations

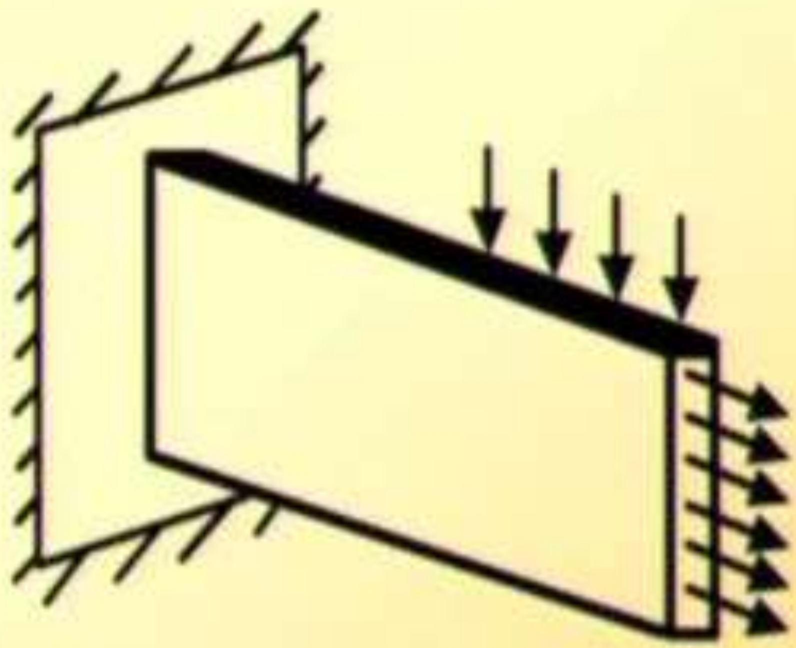
- A general three dimensional elasticity problem involves three dimensional stress state with the components identified as σ_x, σ_y, σ_z, τ_{xz}, τ_{yz}, τ_{xy} where the first three are the normal stresses and the last three are the shear stresses.

- Corresponding to these stress components there are six strain components.

- Two special cases of significant practical importance are the so – called plane stress and plane strain cases.

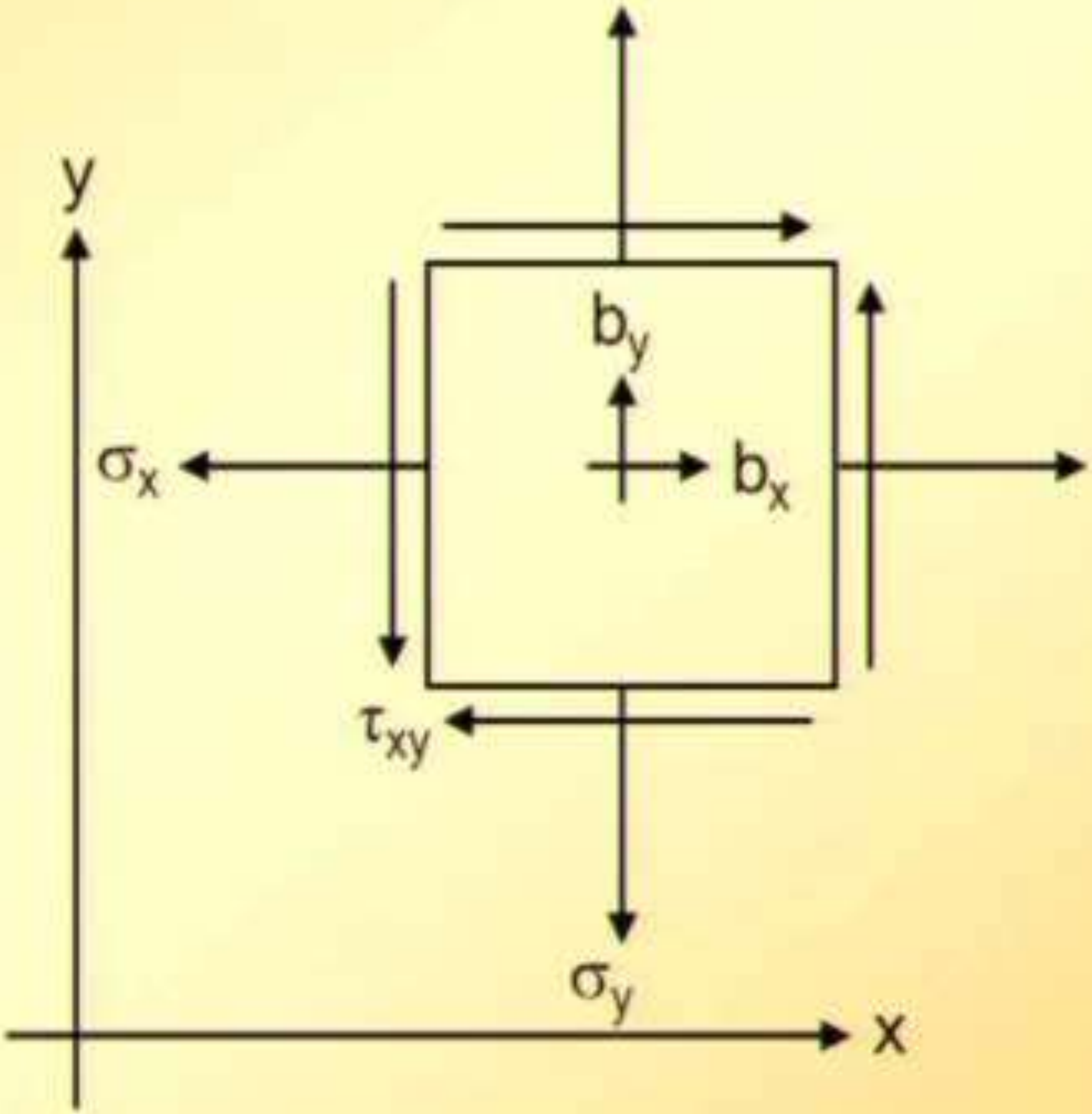
Plane Stress

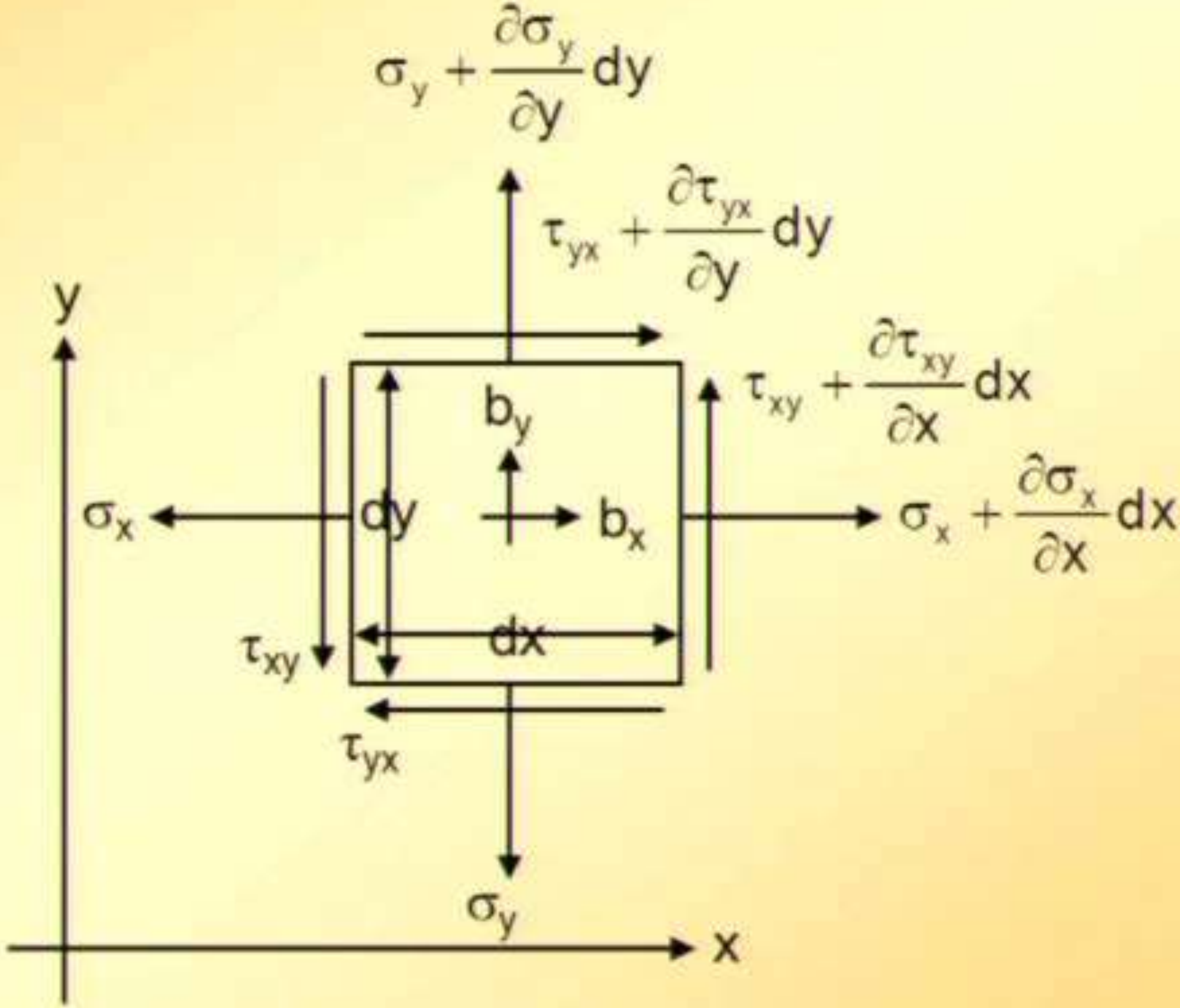
- When analyzing thin plates subjected to in-plane applied forces, it is reasonable to assume that $\sigma_z = \tau_{zx} = \tau_{zy} = 0$.



- The non-zero stress components are: normal stresses σ_x, σ_y and shear stress $\tau_{xy} = \tau_{yx}$.
- The corresponding strain components are: normal strains ε_x, ε_y and shear strain $\gamma_{xy} = \gamma_{yx}$.
- This is known as a plane stress situations.
- Note that the strains ε_z, γ_{yz} and γ_{zx} are not necessarily zero.

- The stress components acting on a differential element are shown in figure below.





$\sigma_y + \frac{\partial \sigma_y}{\partial y} dy$
$\tau_{yx} + \frac{\partial \tau_{yx}}{\partial y} dy$
$\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx$
$\sigma_x + \frac{\partial \sigma_x}{\partial x} dx$
y
b_y
σ_x
dy
b_x
dx
τ_{xy}
τ_{yx}
σ_y
x

The following equilibrium equations can be easily written by considering equilibrium of forces in the x and y directions acting on a differential element.

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + b_y = 0$$

- These equations can be expressed in terms of two displacement components, u (in the x direction) and v (in the y direction).

- Assuming small displacements and strains, the strain-displacement equations are written as follows.

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Assuming linear-elastic material behavior, the stresses and strains are related as follows.

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \qquad \sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right)$$

$$\tau_{xy} = G\gamma_{xy}$$

In matrix notation the stress-strain relationships are written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \dfrac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{or} \quad \sigma = \mathbf{C}_\sigma \varepsilon$$

C_σ is called the constitutive matrix for plane stress.

Substituting the stress-strain relationships into the first equilibrium equations we get

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + b_x = 0$$

$$\frac{E}{1-\nu^2}\left(\frac{\partial \varepsilon_x}{\partial x} + \nu \frac{\partial \varepsilon_y}{\partial x}\right) + G\frac{\partial \gamma_{xy}}{\partial y} = 0$$

Using the strain-displacement equations

$$\frac{E}{1-\nu^2}\left(\frac{\partial^2 u}{\partial x^2} + \nu \frac{\partial^2 v}{\partial x \partial y}\right) + G\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) = 0$$

Rearranging the terms we get

$$\frac{E}{1-\nu^2}\frac{\partial^2 u}{\partial x^2}+G\frac{\partial^2 u}{\partial y^2}=-\left(\frac{E\nu}{1-\nu^2}+G\right)\frac{\partial^2 v}{\partial x\partial y}$$

Multiplying by $(1-\nu^2)/E$ and using the definition of G in terms of E and ν we get

$$\frac{\partial^2 u}{\partial x^2}+\frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2}=-\left(\nu+\frac{1-\nu}{2}\right)\frac{\partial^2 v}{\partial x\partial y}$$

Adding $\partial^2 u/\partial y^2$ to both sides

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial y^2}-\frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2}-\left(\nu+\frac{1-\nu}{2}\right)\frac{\partial^2 v}{\partial x\partial y}$$

Simplifying this gives the first equilibrium equation in terms of displacements as

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1+\nu}{2}\left(\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 v}{\partial x \partial y}\right)$$

Starting from the second equilibrium equation and following similar steps the second equilibrium can be written as:

$$\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{xy}}{\partial x}+b_y=0$$

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{1+\nu}{2}\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x \partial y}\right)$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{1+v}{2}\left(\frac{\partial^2 u}{\partial y^2}-\frac{\partial^2 v}{\partial x \partial y}\right)$$

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{1+v}{2}\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x \partial y}\right)$$

These are second order differential equations.

Note that the two equations are coupled and thus a solution must be obtained by solving the two simultaneously.

Essential boundary conditions:

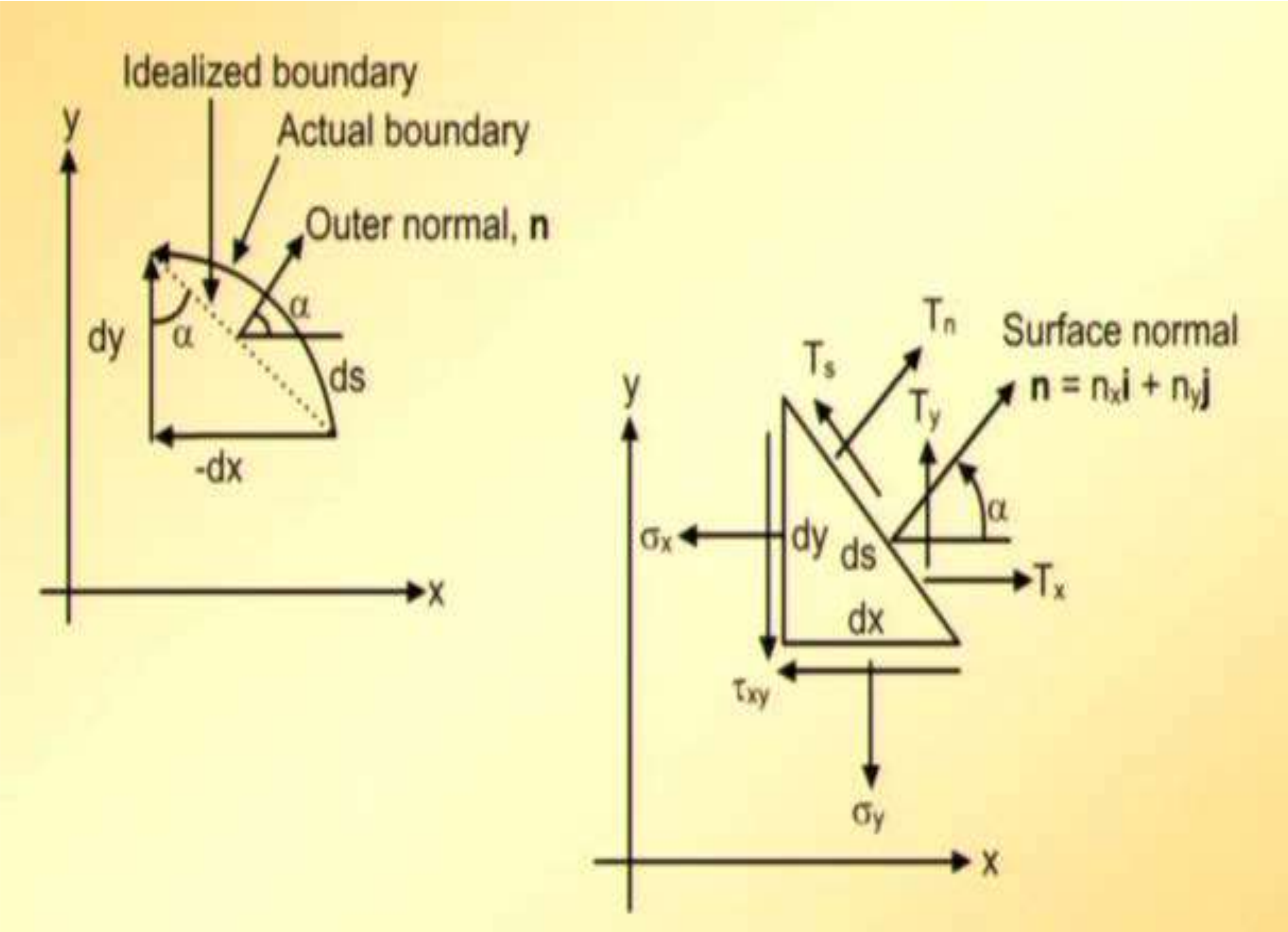
u, v specified on part of boundary

Natural boundary conditions:

Specified surface forces (tractions):

T_x, T_y components in x, y directions

or T_n, T_s normal and tangential components



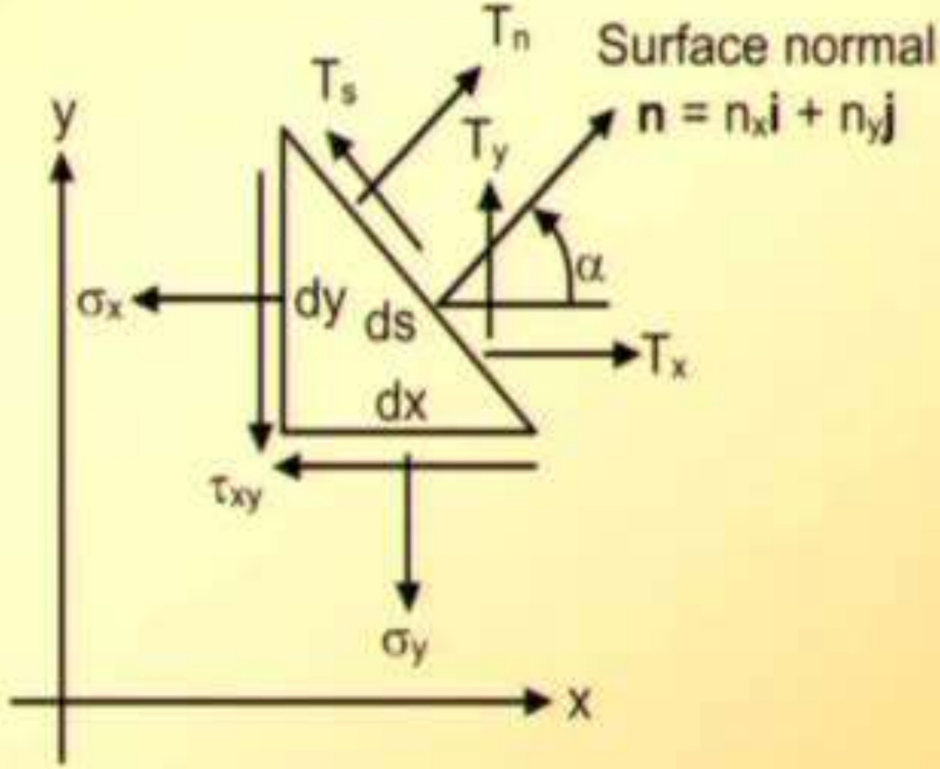
Idealized boundary
Actual boundary
Outer normal, **n**
y
x
dy
-dx
ds
α
α
Surface normal
n = n_x**i** + n_y**j**
T_n
T_s
T_y
T_x
σ_x
σ_y
τ_{xy}
dy
ds
dx
α
y
x

The T_x and T_y components can be obtained by the following transformation.

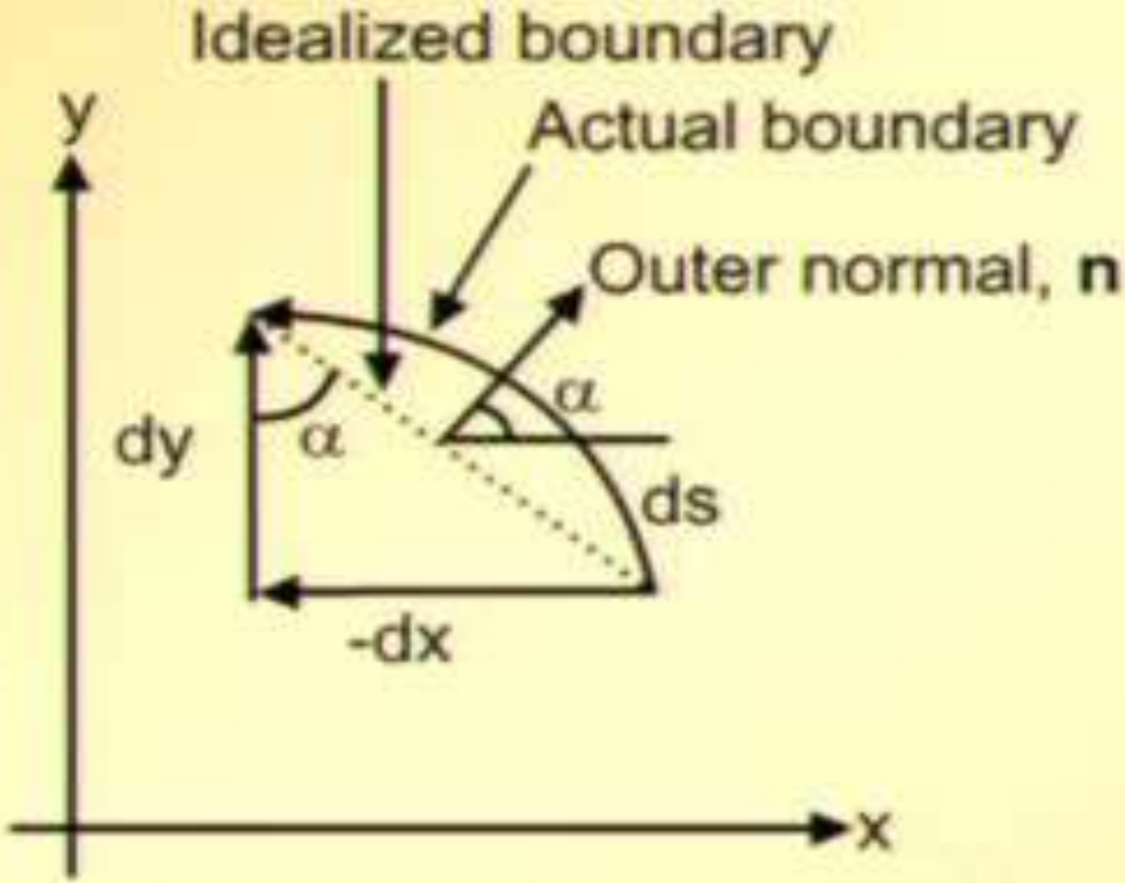
$$T_x = n_x T_n - n_y T_s$$
$$T_y = n_x T_s + n_y T_n$$

or

$$\begin{Bmatrix} T_x \\ T_y \end{Bmatrix} = \begin{Bmatrix} n_x T_n - n_y T_s \\ n_x T_s + n_y T_n \end{Bmatrix}$$
$$= \begin{bmatrix} n_x & -n_y \\ n_y & n_x \end{bmatrix} \begin{Bmatrix} T_n \\ T_s \end{Bmatrix}$$



where n_x and n_y are the direction cosines of the outer surface normal.



$$n_x = \frac{dy}{ds} \qquad n_y = -\frac{dx}{ds}$$

- Considering equilibrium of forces in the x direction we get

$$T_x ds = \sigma_x dy + \tau_{yx} dx$$

or

$$T_x = \sigma_x \frac{dy}{ds} + \tau_{yx} \frac{dx}{ds} = \sigma_x \cos\alpha + \tau_{yx} \sin\alpha$$

Using the stress-strain law

$$T_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right)\cos\alpha + \frac{E}{2(1+\nu)}\gamma_{xy} \sin\alpha$$

$$T_x = \frac{E}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y}\right)\cos\alpha + \frac{E}{2(1+\nu)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\sin\alpha$$

$$T_y = \frac{E}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x}\right)\sin\alpha + \frac{E}{2(1+\nu)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\cos\alpha$$

Thus the applied surface force boundary conditions involve first derivatives $\partial u/\partial x$, $\partial v/\partial x$, $\partial u/\partial y$, $\partial v/\partial y$ and hence are natural boundary conditions for a second order boundary value problem.

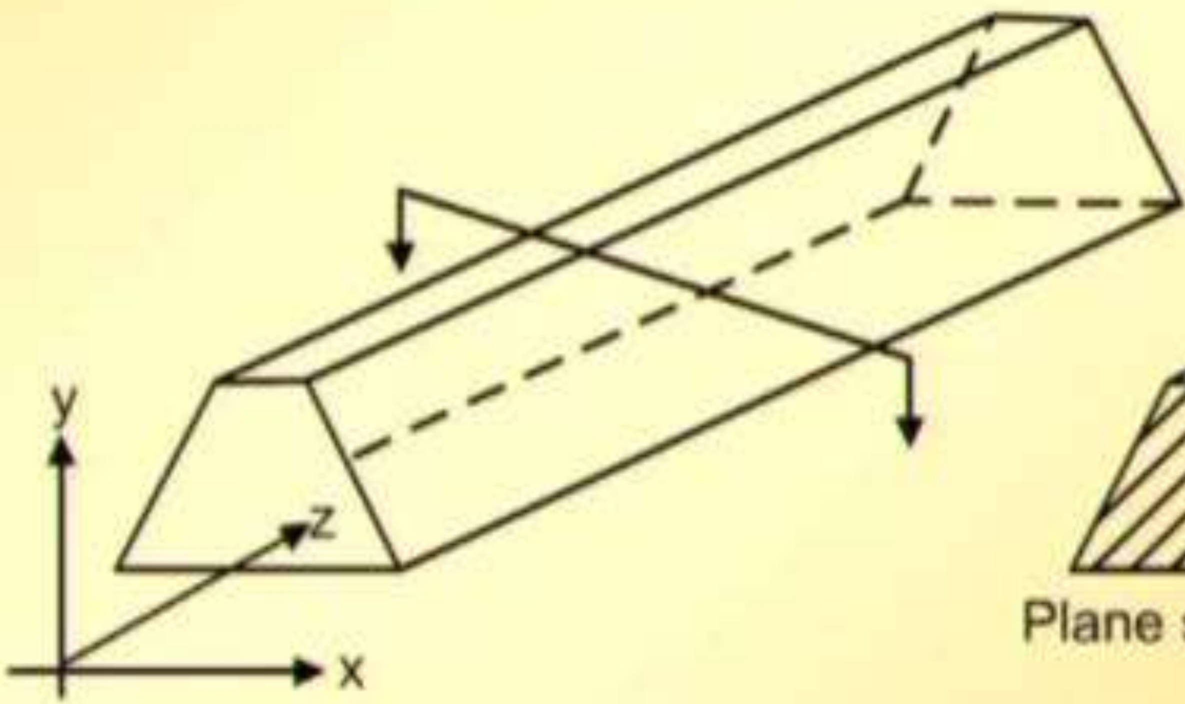
Plane Strain

- In a plane strain problem it is assumed that

$$\varepsilon_z = \gamma_{zx} = \gamma_{zy} = 0.$$

- This is a reasonable assumption when analyzing systems which are much longer in one dimension than the others, such as dams.

- As illustrated in figure below, for these systems the end effects may be neglected and therefore a unit "slice" can be modeled as a plane strain problem.



Plane strain model

- The non zero strain components are: normal strains ε_x, ε_y and shear strain $\gamma_{xy} = \gamma_{yx}$.
- The corresponding stress components are: normal stresses σ_x, σ_y and shear stress $\tau_{xy} = \tau_{yx}$.
- Note that the stress components, σ_z, τ_{yz} and τ_{zx} are not necessarily zero.

- A plane strain problem is formulated in essentially the same way as a plane stress.

- The only difference in the two situations is the constitutive equation relating stresses with the strains.

- In a plane strain problem, the stresses are related to strains through the following equations.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad \text{or} \quad \sigma = \mathbf{C}_\varepsilon \varepsilon$$

$\mathbf{C}_\varepsilon$ is called the constitutive matrix for plane strain.

Equivalence between Plane Stress and Plane Strain Problems

- The constitutive equations are the only difference between the plane stress and plane strain formulations.

- By defining equivalent values for E and ν it is possible to easily move from one formulation to the other.

- Solving plane stress problem when plane strain formulation is known

$$\text{Replace E by } E\left[1-\left(\frac{\nu}{1+\nu}\right)^2\right] \text{ and } \nu \text{ by } \frac{\nu}{1+\nu}$$

- Solving plane strain problem when plane stress formulation is known

$$\text{Replace E by } \frac{E}{1-\left(\frac{\nu}{1-\nu}\right)^2} \text{ and } \nu \text{ by } \frac{\nu}{1-\nu}$$